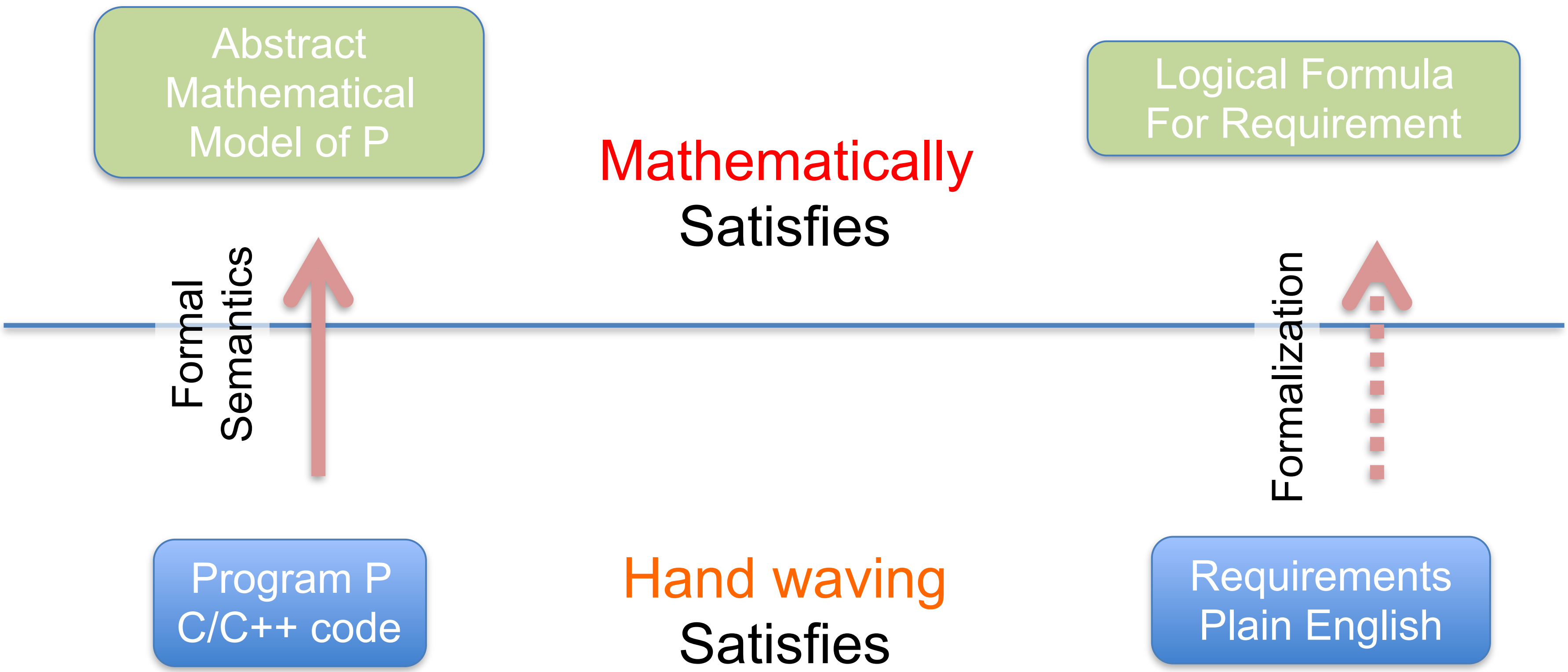
Algorithmic Verification COMP 3153

Lecture 15

Symbolic Model Checking – 1

(last update: April 6, 2017)

The Big Picture



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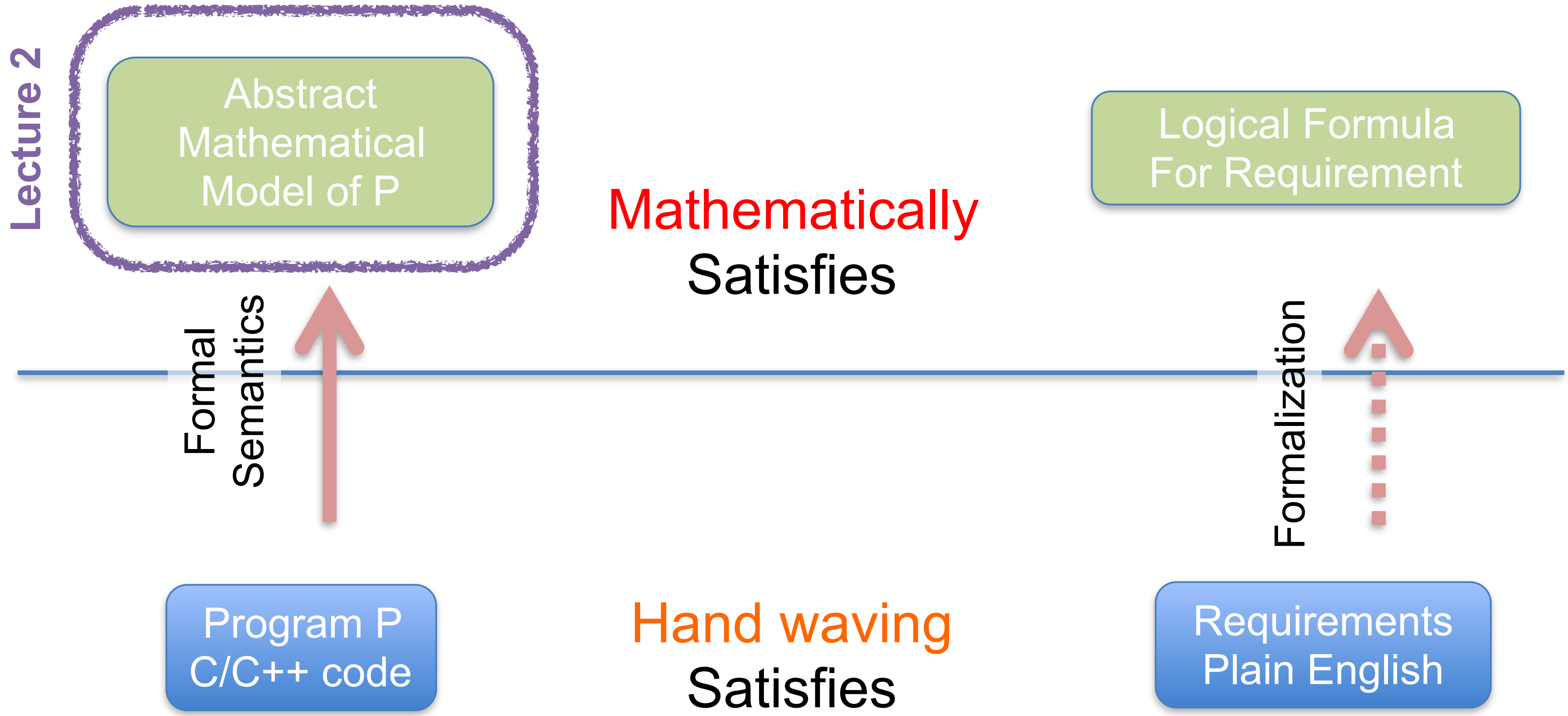

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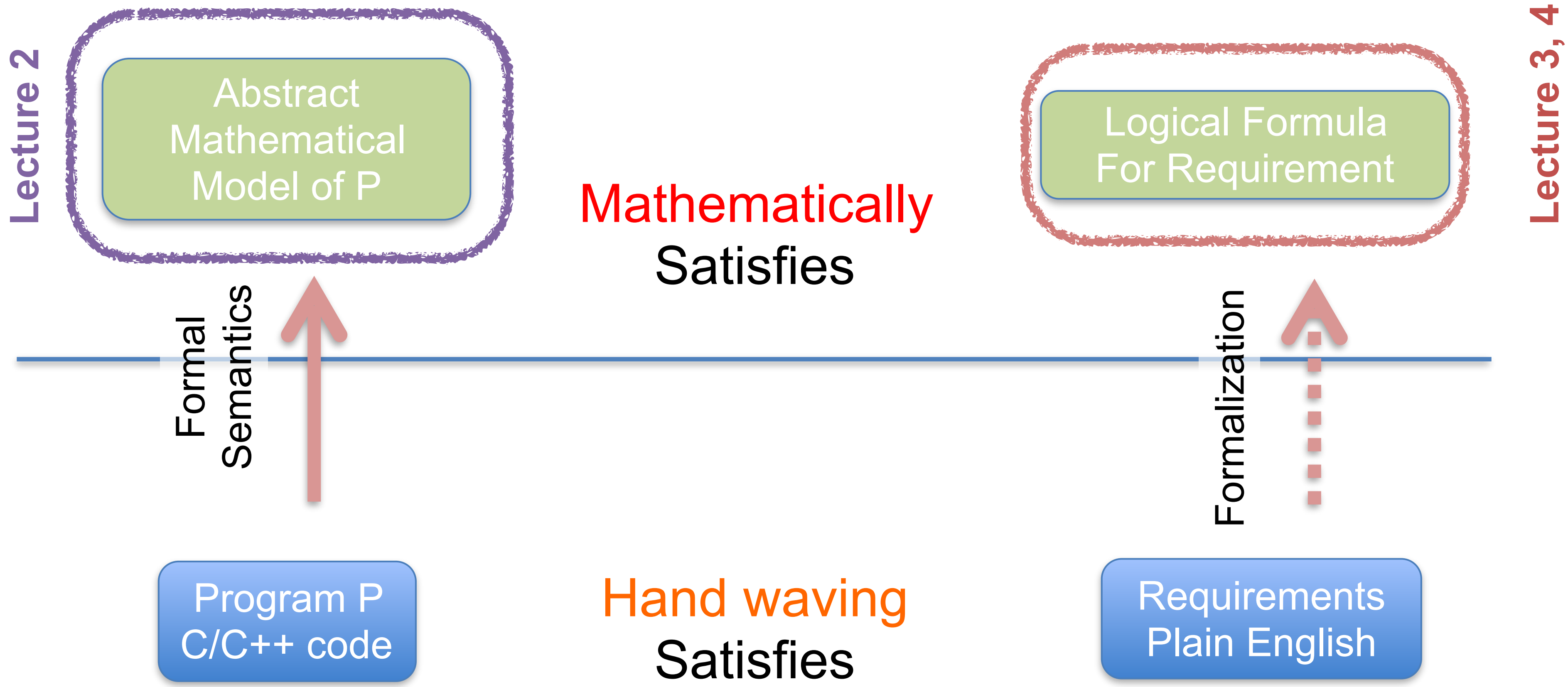

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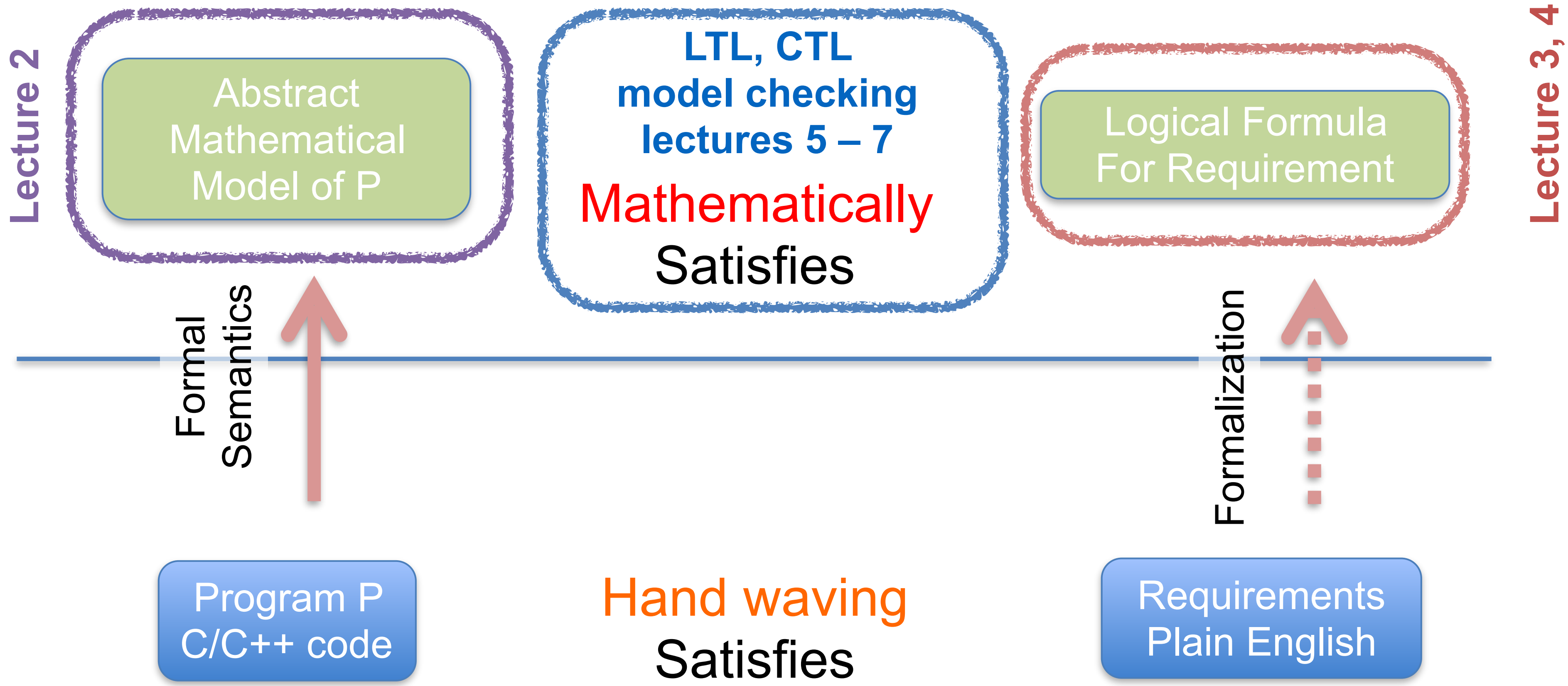

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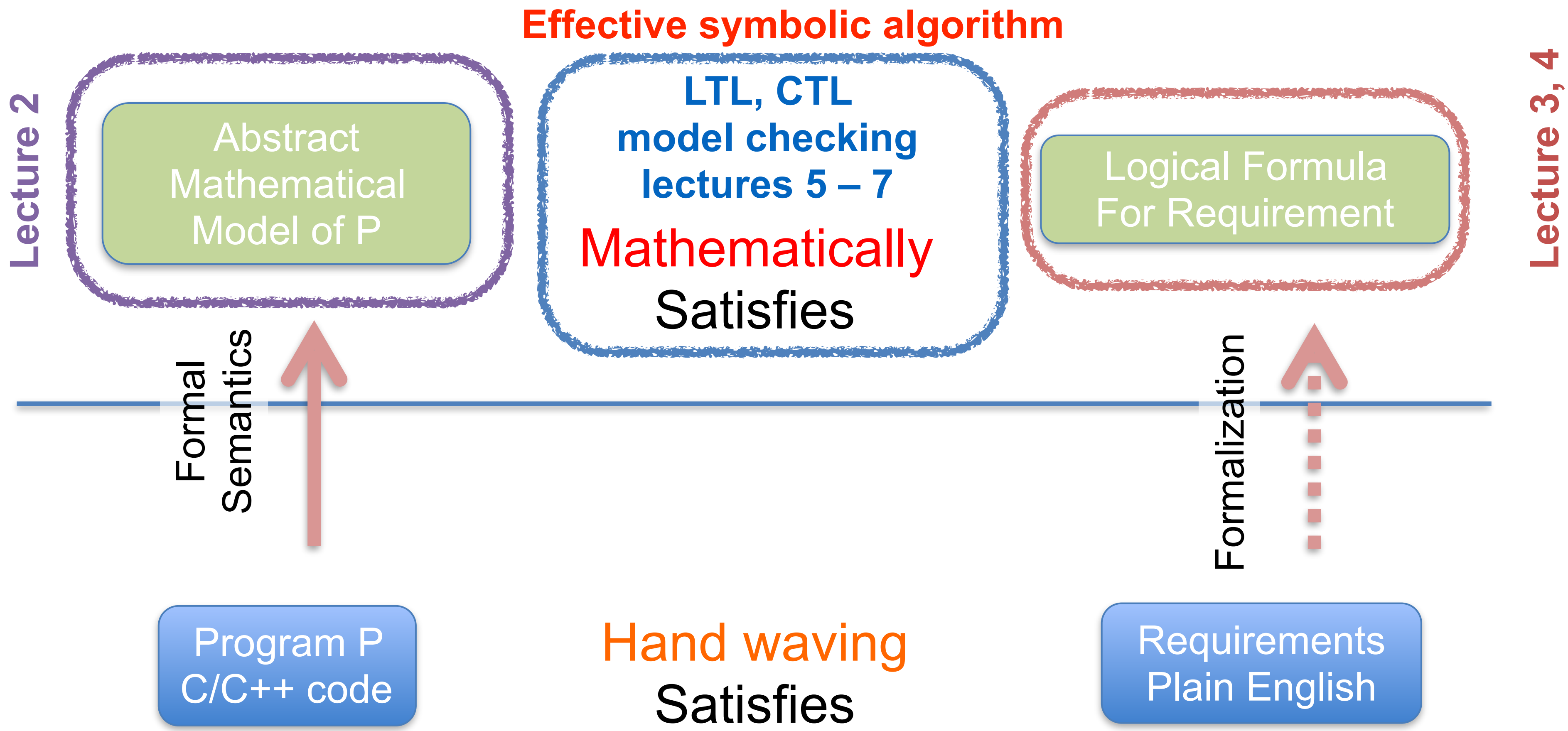


Content for this lecture

1. Lattice theory and fix points
2. Fix point characterisation of CTL
3. Symbolic fix point algorithm for CTL

Fix Points

Complete Lattices and Fix Points

Partially Ordered Set (poset)

$(L, \leq)$ a poset if $\leq$:

- reflexive, $x \leq x$
- anti-symmetric, $x \leq y \wedge y \leq x \implies x = y$
- transitive, $x \leq y \wedge y \leq z \implies x \leq z$

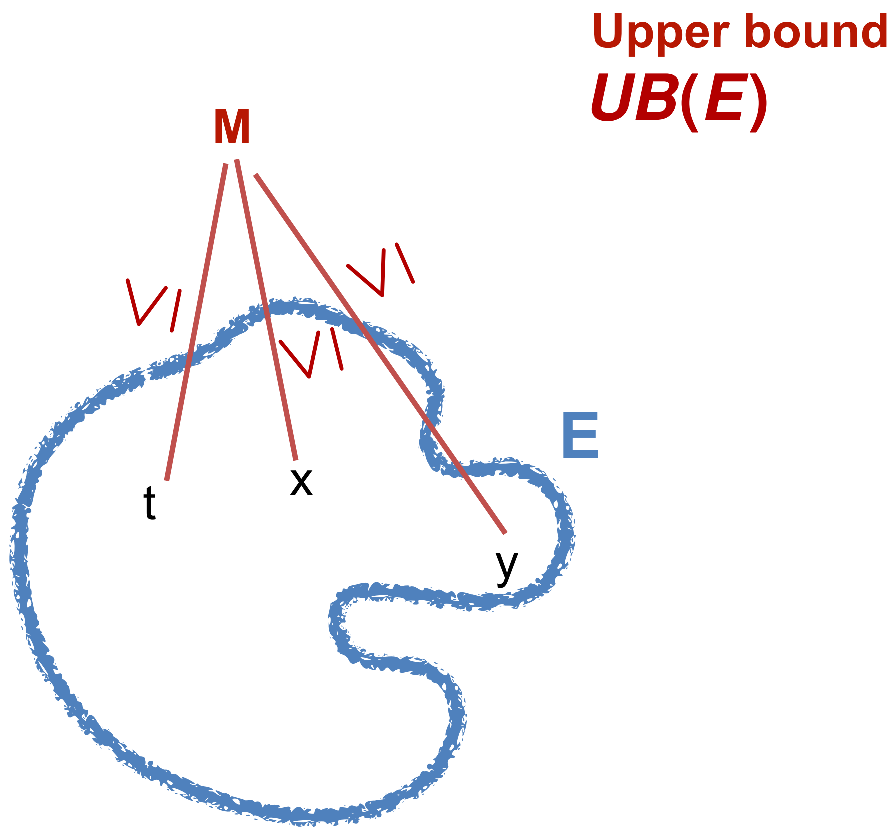
Examples

$(\mathcal{P}(X), \subseteq)$

$(\mathbb{Z}, \leq)$

Upper/Lower Bounds

Upper/Lower Bounds



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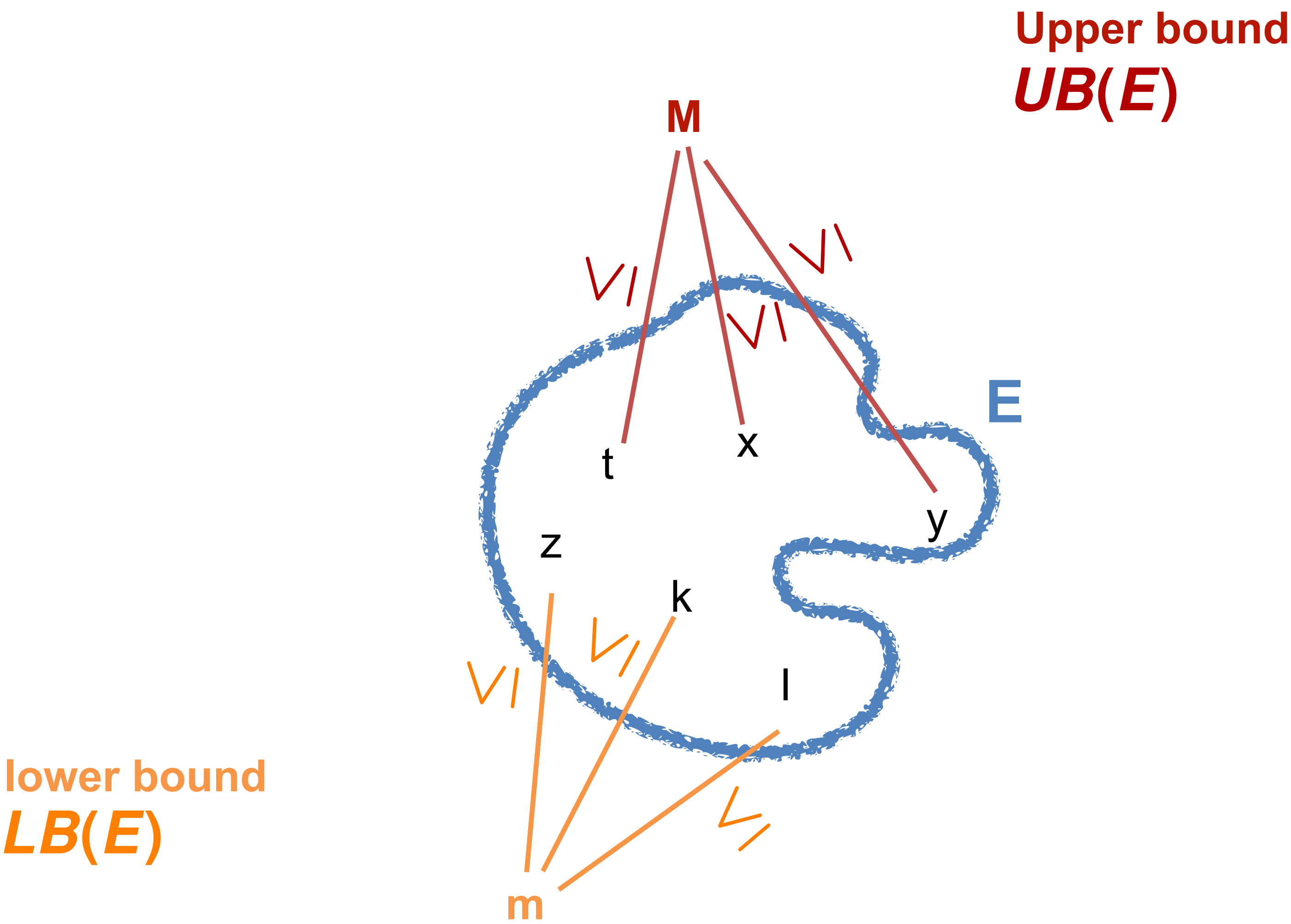


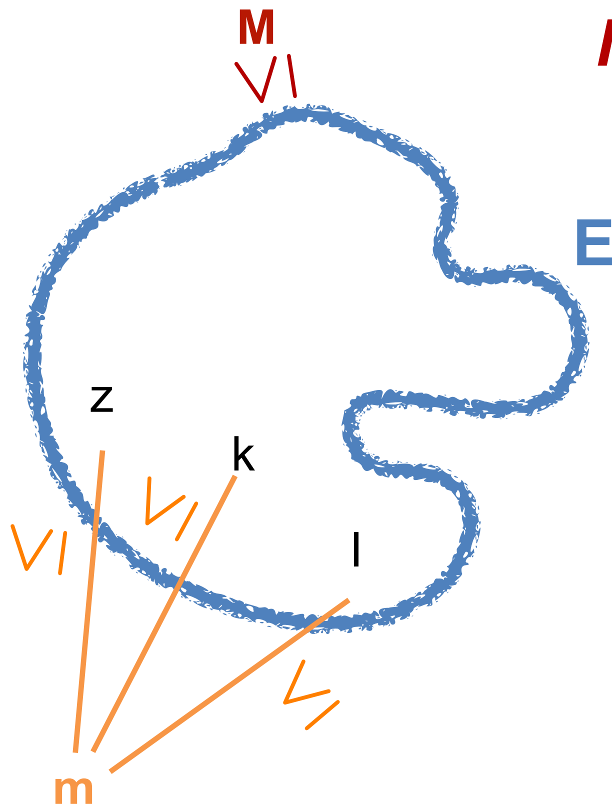
Upper/Lower Bounds

Upper bound

$\boldsymbol{UB(E)}$

Least upper bound

$\boldsymbol{M \in UB(E) \wedge M \leq UB(E)}$



lower bound

$\boldsymbol{LB(E)}$

Upper/Lower Bounds

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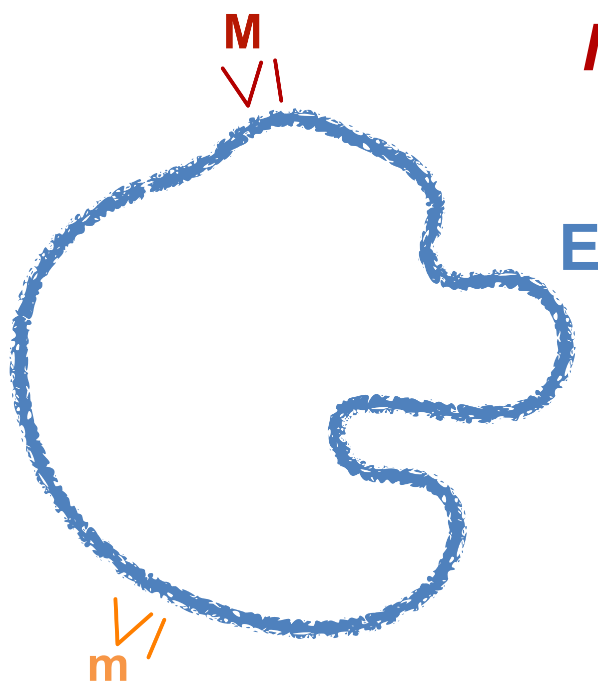

lower bound

$LB(E)$

greatest lower bound

$m \in LB(E) \wedge LB(E) \leq m$

Upper/Lower Bounds

LUB and GLB unique (when they exist)

Upper bound

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Least upper bound

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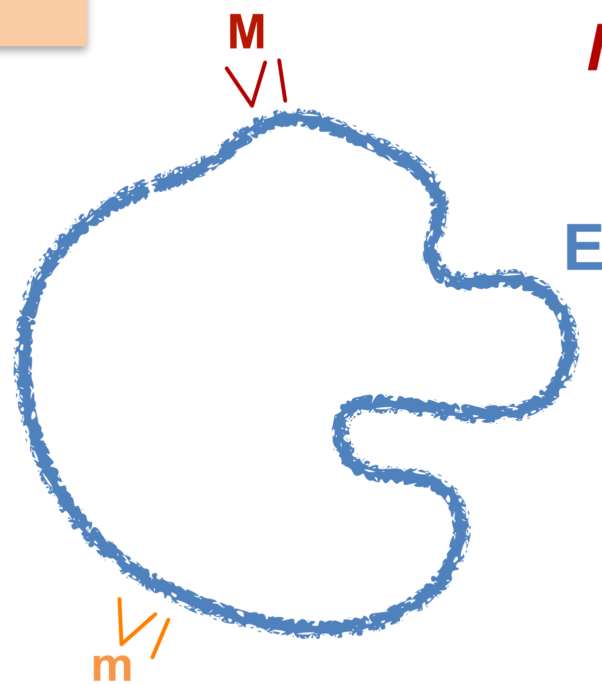

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(Complete) Lattice

(L, $\leq$) is a lattice iff every pair **(x, x')** of ***L*** has a LUB and GLB

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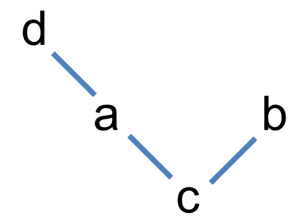

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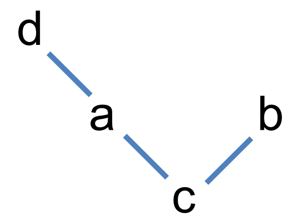

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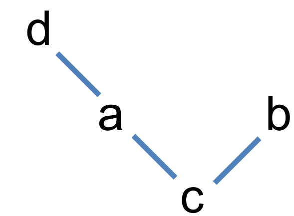

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Special elements: $LUB(L) = \top$ and $GLB(L) = \bot$

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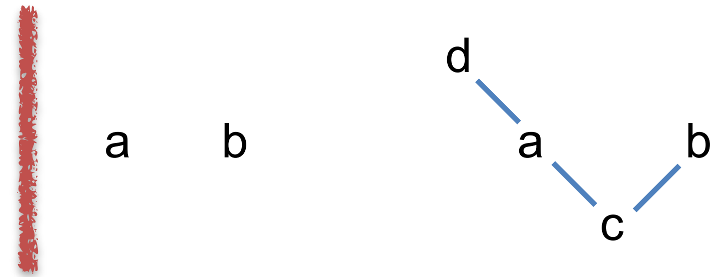

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$(\mathbb{Z} \cup \{-\infty, +\infty\}, \leq)$

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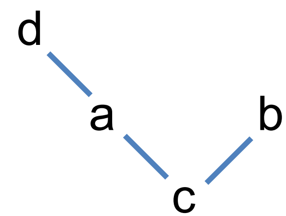

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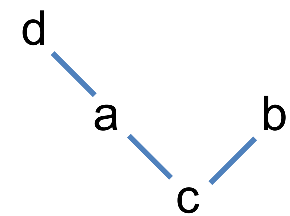

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$(\mathbb{Z}, \leq)$

$(\mathbb{Q}, \leq)$

Fix Points and Complete Lattices

$(L, \leq)$ poset

$f : L \to L$ is monotonic if $x \leq y \implies f(x) \leq f(y)$

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Knaster-Tarski Theorem:

f is monotonic and $(L, \leq)$ is a complete lattice then

f has a least and a greatest fix point.

Properties of Fix Points

$lfp(f) = GLB(\{x \in L, f(x) \leq x\})$

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1. $X = \{x \in L, f(x) \leq x\}$, $m = GLB(X)$
2. $\top \geq f(\top)$ and $\top \in X$
3. f monotonic: $\forall x \in X, f(m) \leq f(x) \leq x$
4. $f(m) \in LB(X)$ and $f(m) \leq m$
5. f monotonic: $f(f(m)) \leq f(m)$ and $f(m) \in X$
6. $m \in LB(X)$ and $m \leq f(m)$
7. $z = f(z)$ then $z \in X$ and $m \leq z$

Properties of Fix Points

$lfp(f) = GLB(\{x \in L, f(x) \leq x\})$ $gfp(f) = LUB(\{x \in L, x \leq f(x)\})$

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Computation of Fix Points on Finite Sets

$(L, \leq)$ a finite lattice

$f : L \to L$ is monotonic

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$f : L \rightarrow L$ is monotonic

$\bot = GLB(L)$ and $\top = LUB(L)$

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$\bot$	$f(\bot)$	$f(f(\bot))$		$f^k(\bot)$		$f^l(\bot)$		

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$\boldsymbol{f} : \boldsymbol{L} \to \boldsymbol{L}$ is monotonic

$\bot = \boldsymbol{GLB}(\boldsymbol{L})$ and $\top = \boldsymbol{LUB}(\boldsymbol{L})$

$\exists \boldsymbol{k} \leq |\boldsymbol{L}|, \boldsymbol{lfp}(\boldsymbol{f}) = \boldsymbol{f}^{\boldsymbol{k}}(\bot)$

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$\leq \quad \leq \quad \leq \quad \leq \quad \leq \quad \leq$

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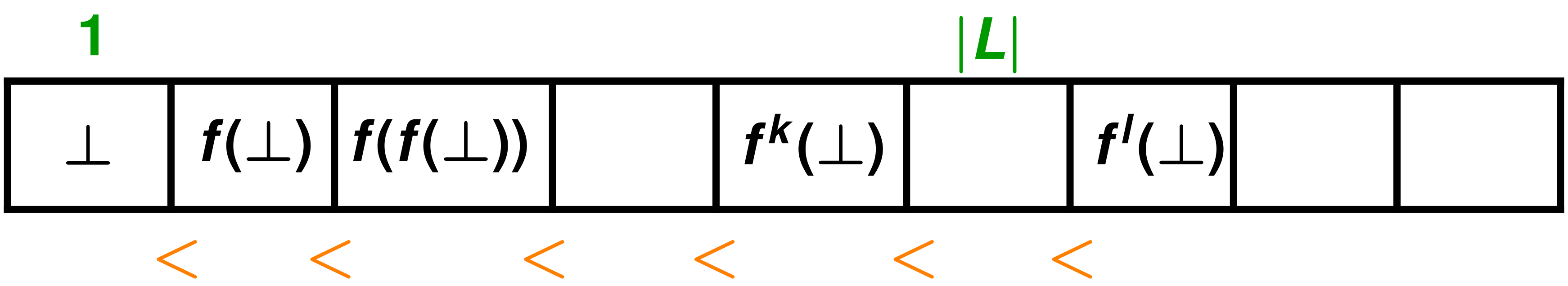

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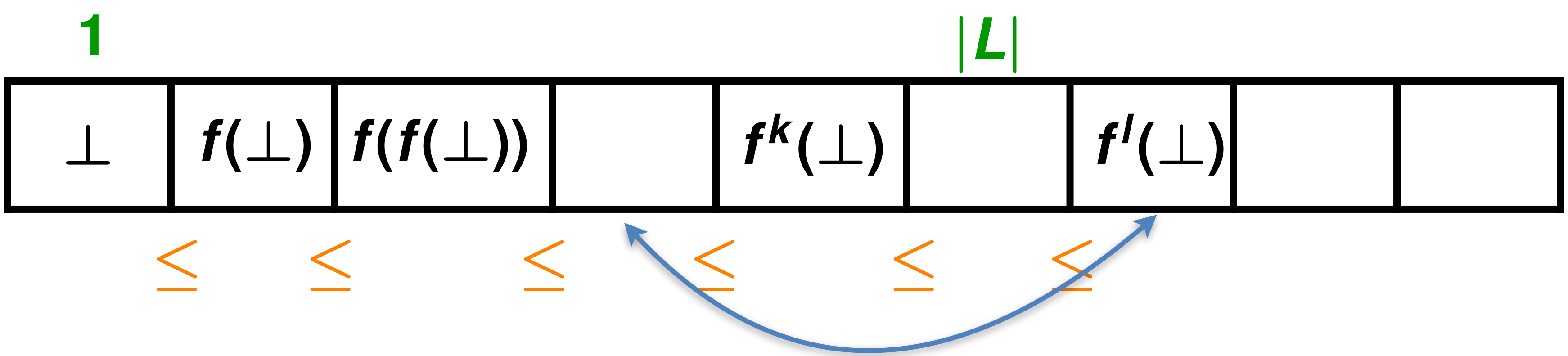

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y

$\geq$

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$\vee\!| \quad \vee\!|$

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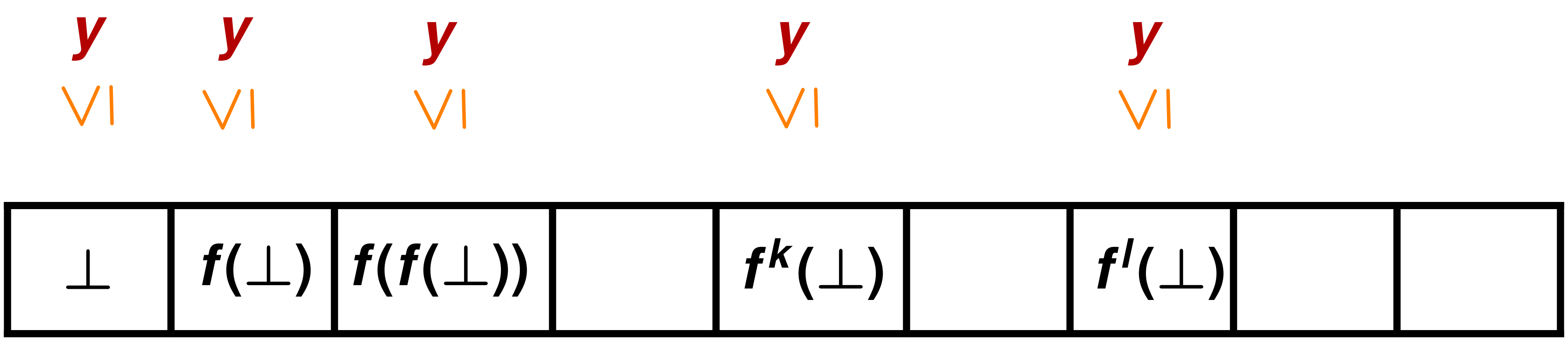

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Algorithm to Compute LFP/GFP

Procedure ComputeFixPoint($f : E \to E, \iota \in \{\bot, \top\}$)

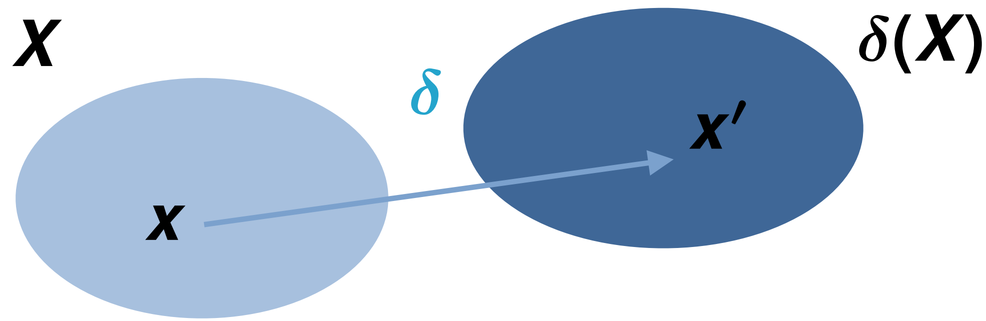
```
/* f is monotonic */
/* E finite and ⊥ = GLB(E), ⊤ = LUB(E) */
P := ι;
N := f(P);
while P ≠ N do
    P := N;
    N := f(N);
return N;
```

$lfp(f)$ = ComputeFixPoint($f, \bot$) $gfp(f)$ = ComputeFixPoint($f, \top$)

Successors/Predecessors

A labeled automaton $\boldsymbol{A = (Q, q_0, \Sigma, \delta, L)}$

$$\delta(X) = \bigcup_{x \in X} \{x' \mid x' \in \delta(x)\}$$



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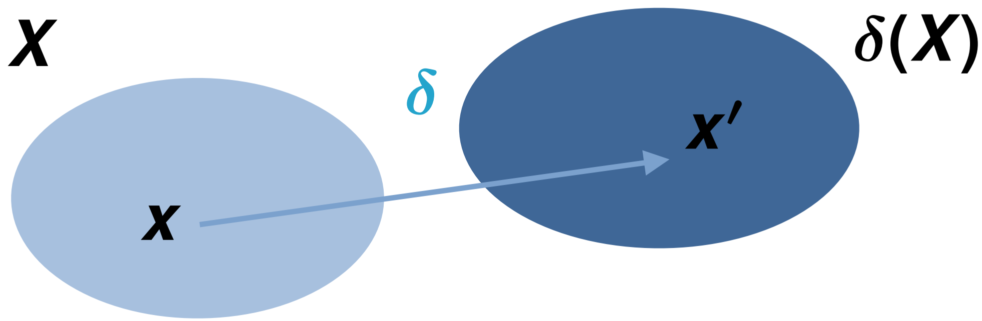
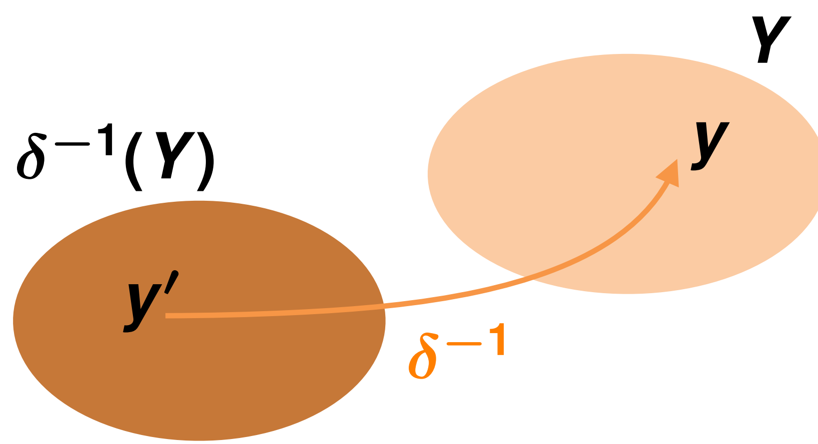
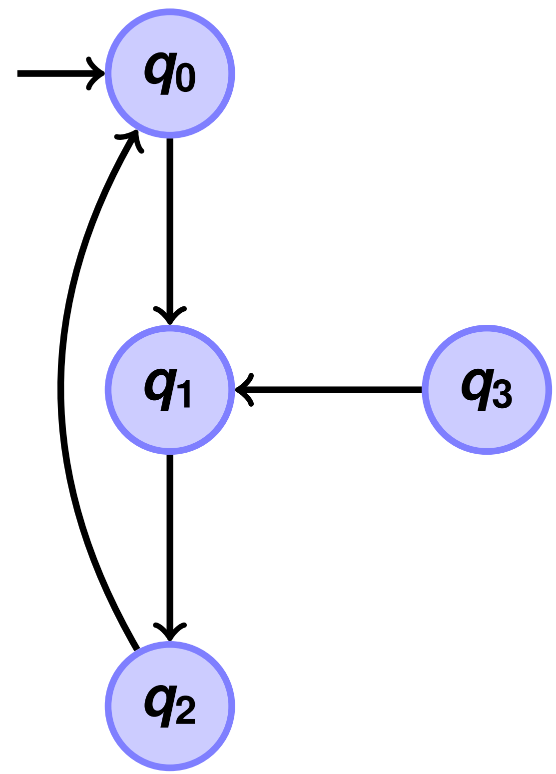


$$\delta^{-1}(Y) = \bigcup_{y \in Y} \{y' \mid y \in \delta(y')\}$$

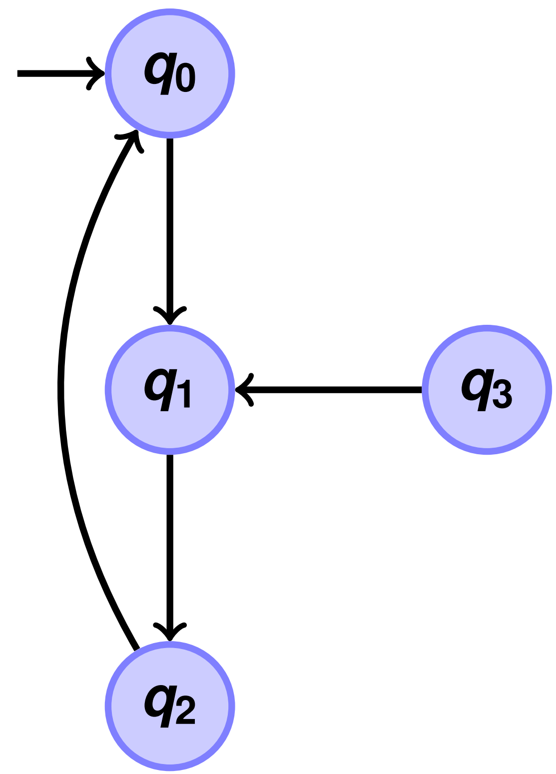


Example – 1



Example – 1

$\boldsymbol{f(S) = (\{q_0\} \cup S) \cup \delta(S)}$



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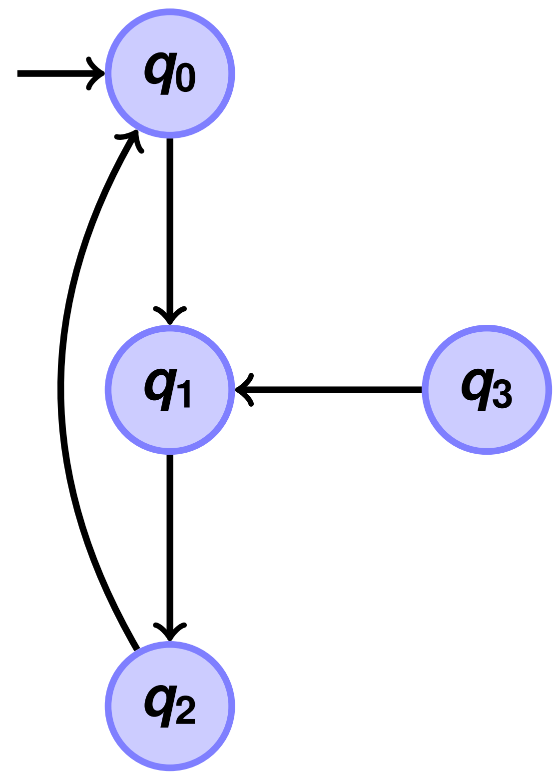
Reach($\{q_0\}$) = $lfp(f)$



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1. Reach($\{q_0\}$) is a fix point

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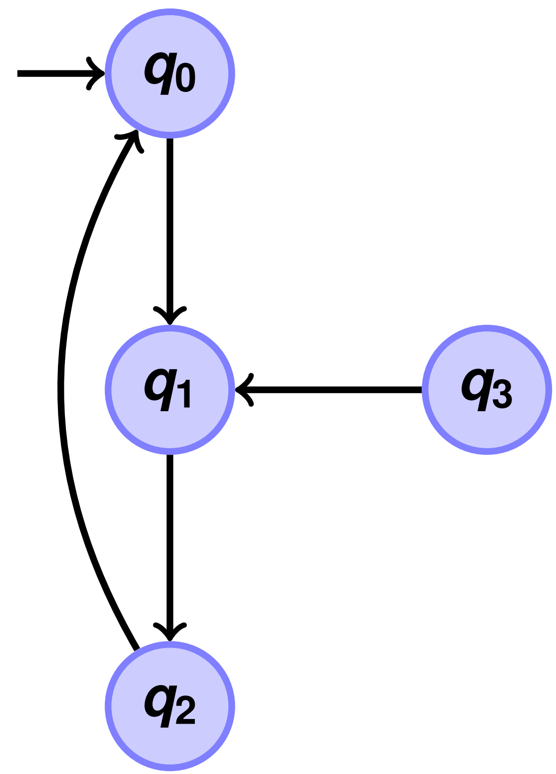


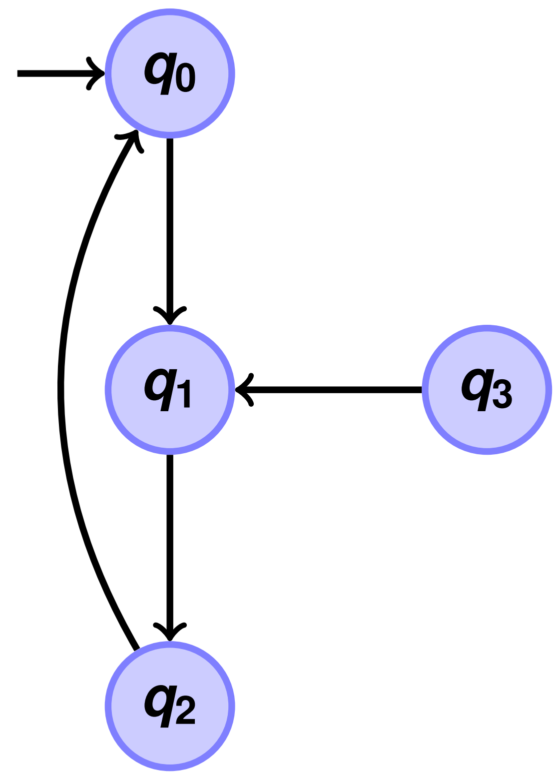
Example – 1

$\boldsymbol{f}(\boldsymbol{S}) = (\{\boldsymbol{q}_0\} \cup \boldsymbol{S}) \cup \delta(\boldsymbol{S})$

1. Reach($\{\boldsymbol{q}_0\}$) is a fix point

   Reach($\boldsymbol{q}_0$) $\subseteq$ $\boldsymbol{f}$(Reach($\boldsymbol{q}_0$))

Reach($\{\boldsymbol{q}_0\}$) = $\boldsymbol{lfp}(\boldsymbol{f})$



Example – 1

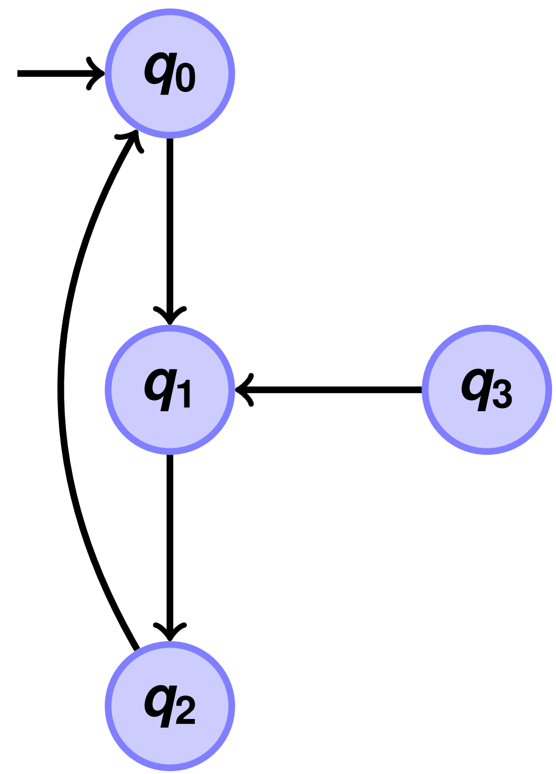
$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

1. Reach($\{q_0\}$) is a fix point

   Reach(q_0) $\subseteq$ f(Reach(q_0))

   $q \in f(\text{Reach}(q_0))$: $q = q_0$
   or $q \in \text{Reach}(q_0)$
   or $q \in \delta(\text{Reach}(q_0))$

Reach($\{q_0\}$) = $lfp(f)$



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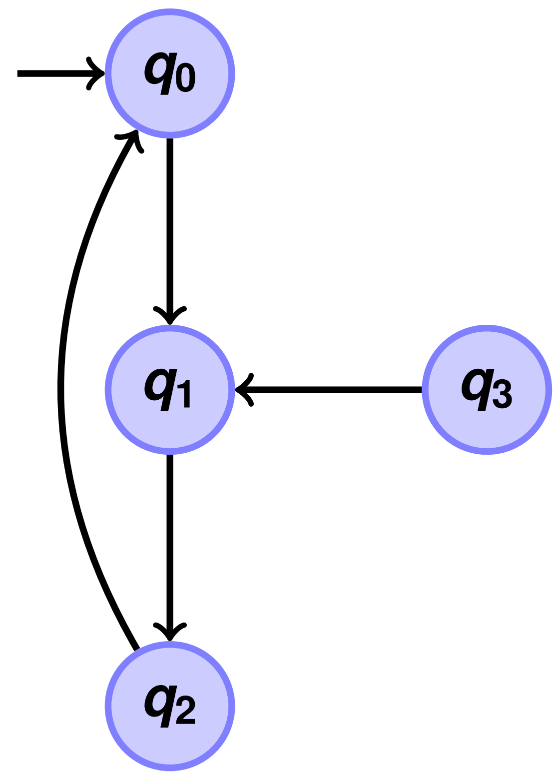
   or $\boldsymbol{q} \in$ Reach($\boldsymbol{q}_0$)
   or $\boldsymbol{q} \in \delta$(Reach($\boldsymbol{q}_0$))

2. Reach($\{\boldsymbol{q}_0\}$) is the least fix point



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   Reach(q_0) $\subseteq$ f(Reach(q_0))

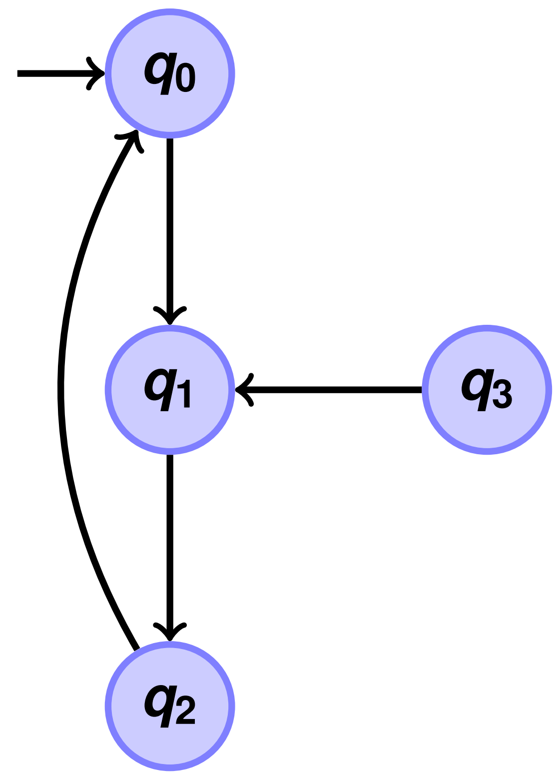
   $q \in f$(Reach(q_0)): $q = q_0$
   or $q \in$ Reach(q_0)
   or $q \in \delta$(Reach(q_0))

2. Reach($\{q_0\}$) is the least fix point

   $Y = f(Y)$

   $q \in$ Reach(q_0): $q_0 \xrightarrow{n} q$

   Prove: $\forall n$, if $q_0 \xrightarrow{n} q$ then $q \in Y$



Example – 1

$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

Reach($\{q_0\}$) = $lfp(f)$

1. Reach($\{q_0\}$) is a fix point

   Reach(q_0) $\subseteq$ f(Reach(q_0))

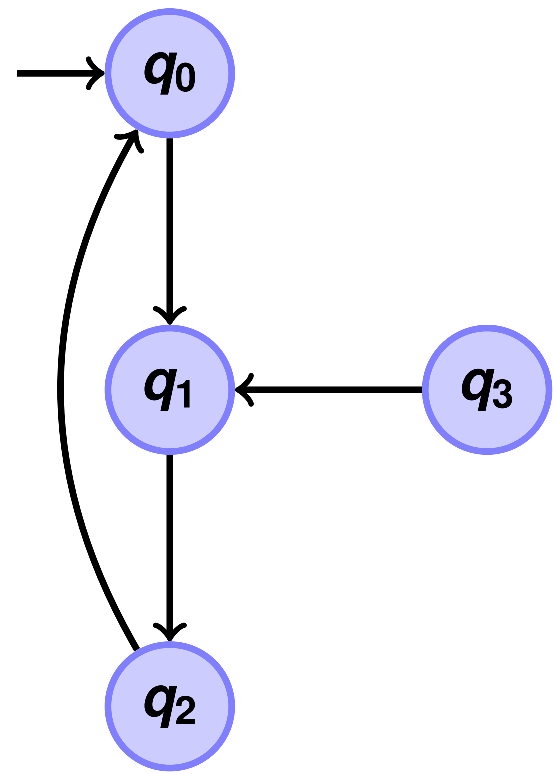
   $q \in f$(Reach(q_0)): $q = q_0$
   or $q \in$ Reach(q_0)
   or $q \in \delta$(Reach(q_0))

2. Reach($\{q_0\}$) is the least fix point

   $Y = f(Y)$

   $q \in$ Reach(q_0): $q_0 \xrightarrow{n} q$

   Prove: $\forall n$, if $q_0 \xrightarrow{n} q$ then $q \in Y$

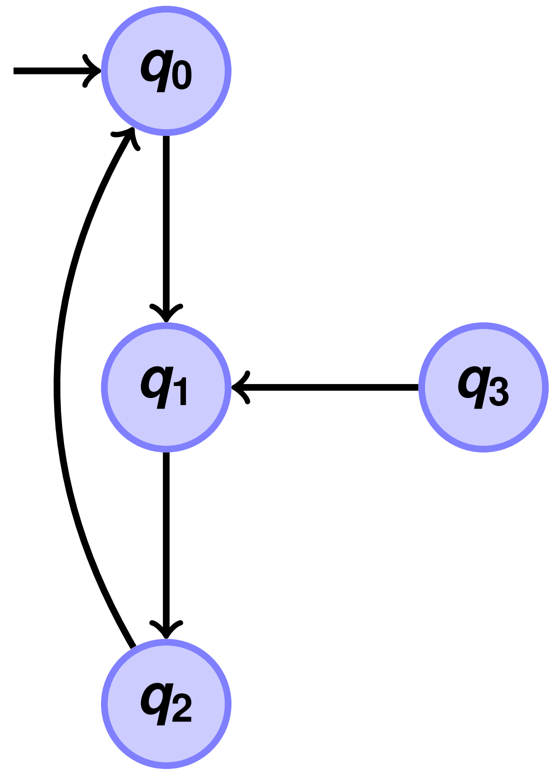


0. Reach($\{q_0\}$) is NOT the greatest fix point

Example – 1

$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

Reach($\{q_0\}$) = $lfp(f)$



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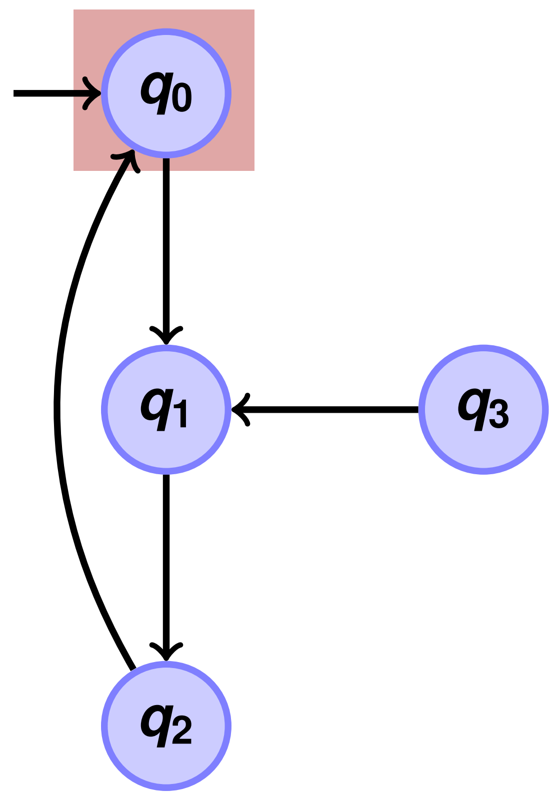
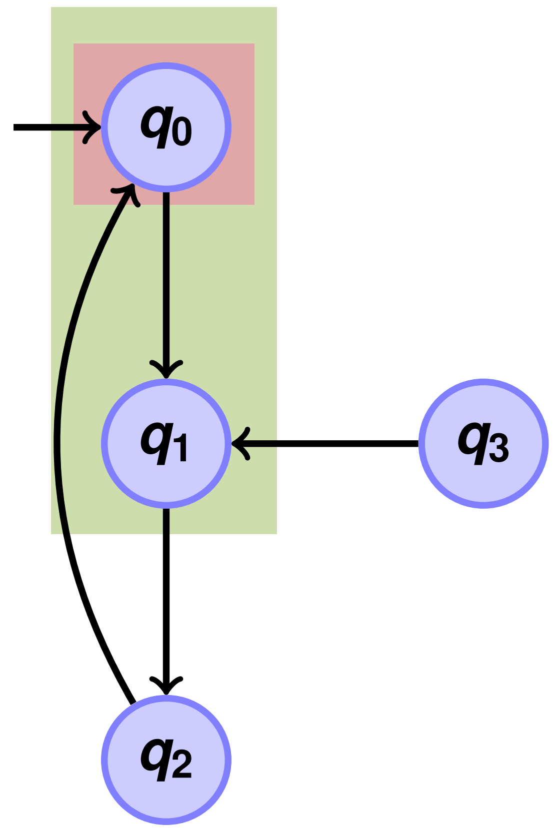


Example – 1

$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

$\text{Reach}(\{q_0\}) = \mathit{lfp}(f)$



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$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

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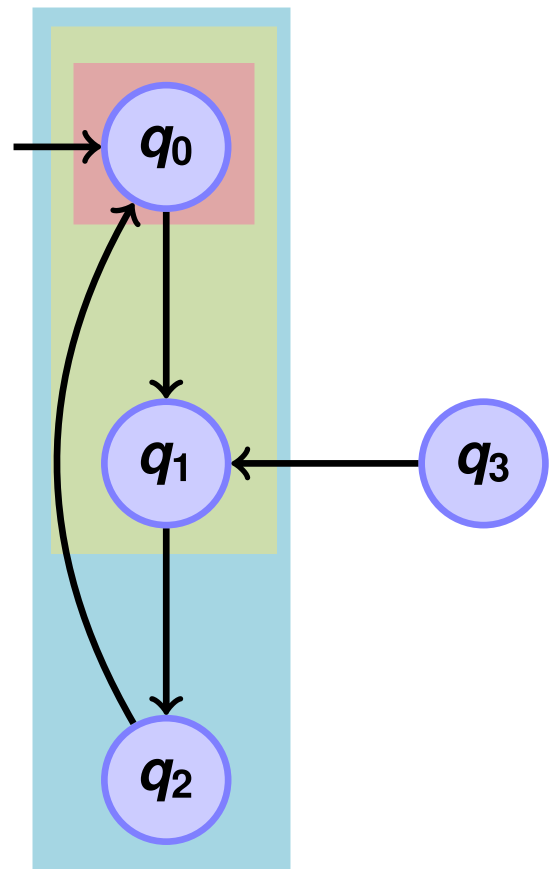


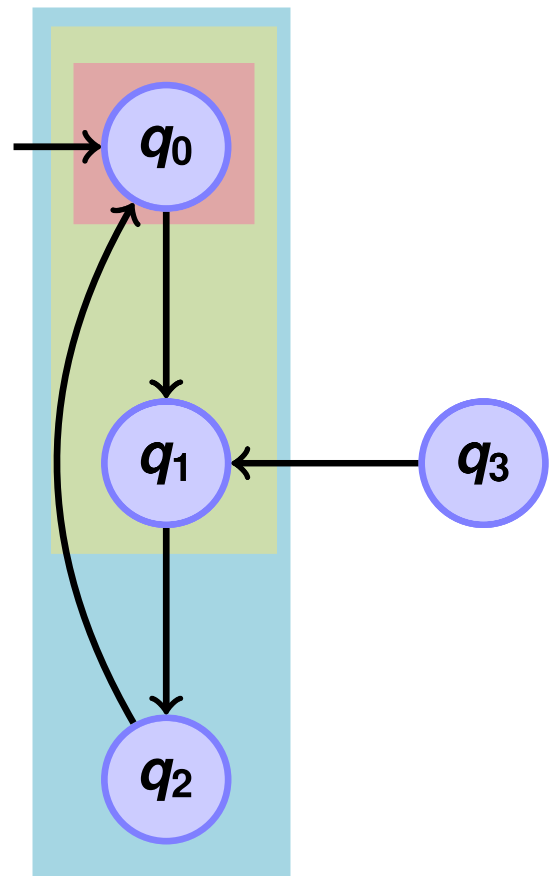
Example – 1

$f(S) = (\{q_0\} \cup S) \cup \delta(S)$

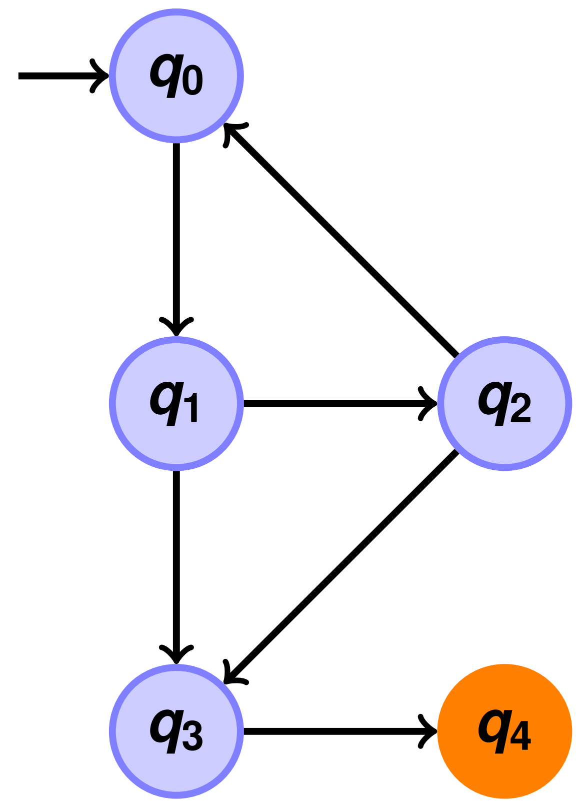
Reach($\{q_0\}$) = $lfp(f)$

$f^k(\varnothing)$

$S_0 = \varnothing$
$S_1 = f(S_0) = \{q_0\}$
$S_2 = f(S_1) = \{q_0, q_1\}$
$S_3 = f(S_2) = \{q_0, q_1, q_2\}$
$S_4 = f(S_3) = \{q_0, q_1, q_2\} = S_3$

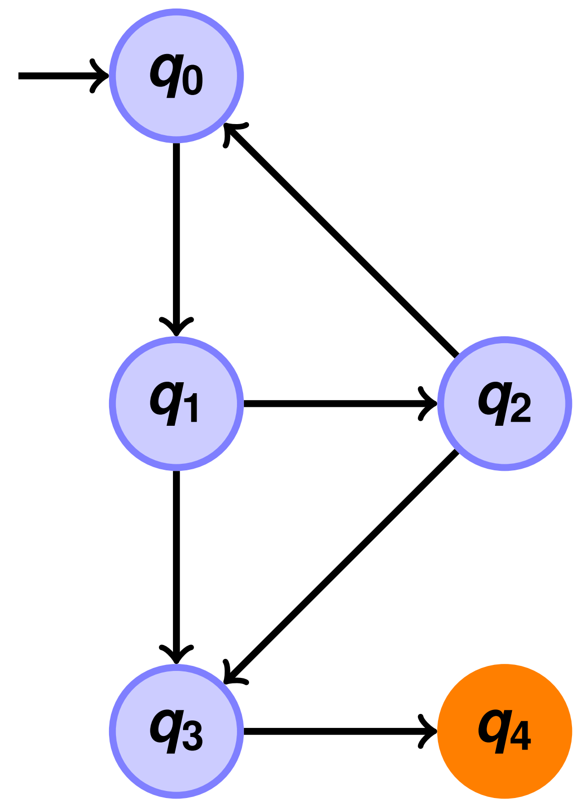


Example – 2



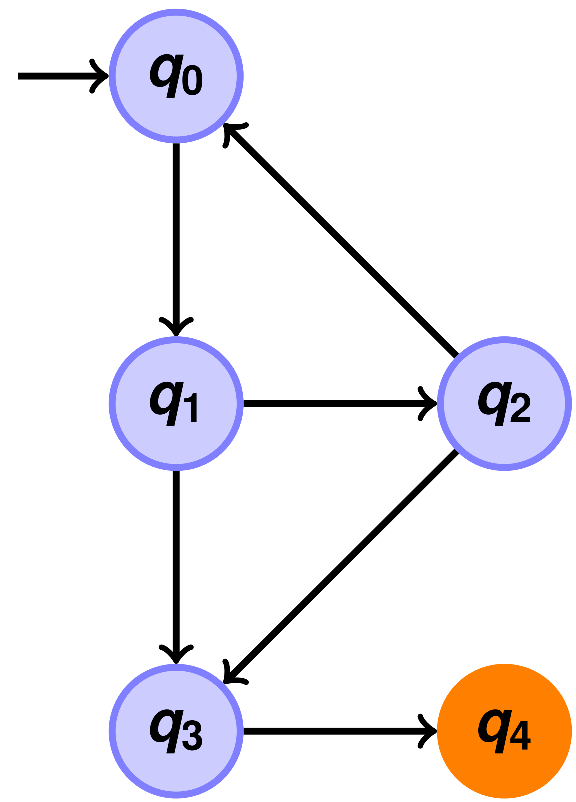
Example – 2

$K = \{q \mid q \models \mathrm{E\,G} \neg$ ● $\}$



Example – 2

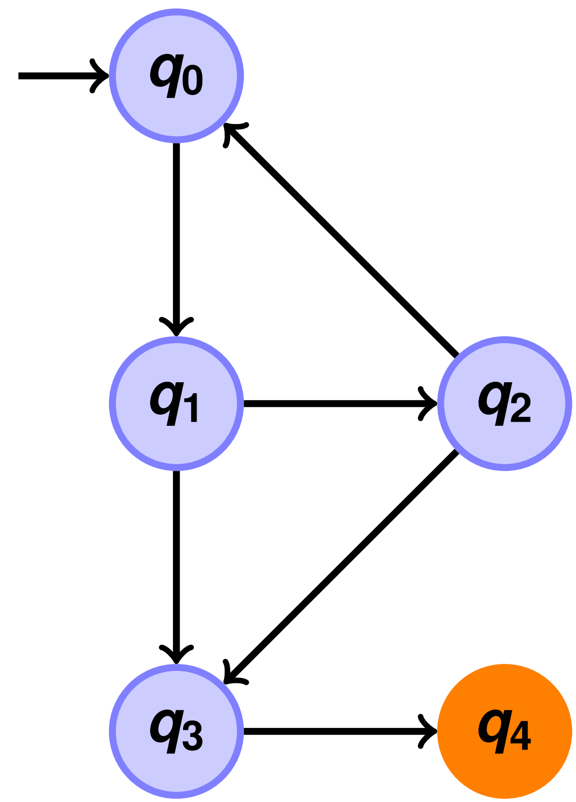
$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$ $K = \{q \mid q \models \text{E G}\neg$ ●$\}$



Example – 2

$f(S) = (\neg \bullet) \cap \delta^{-1}(S)$ $\quad$ $K = \{q \mid q \models \mathrm{E\,G}\neg \bullet\}$

$K = gfp(f)$



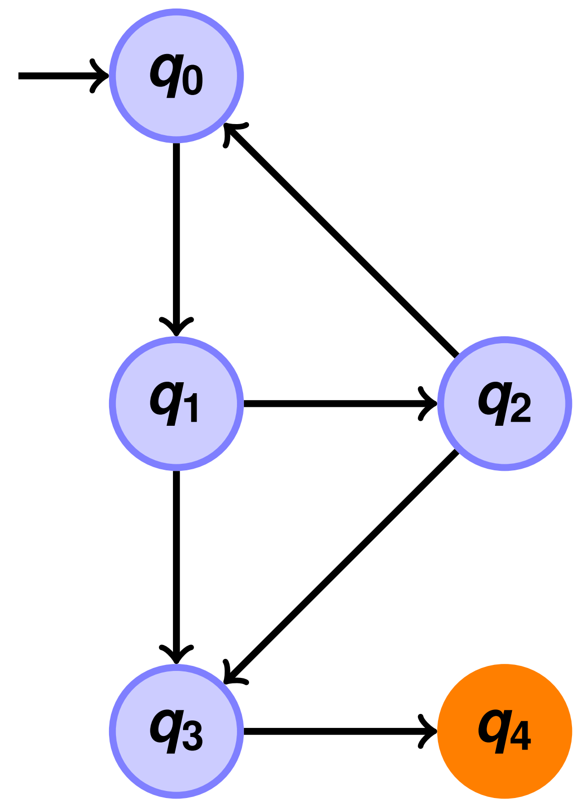
Example – 2

$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

$K = \{q \mid q \models \mathsf{E}\,\mathsf{G}\neg$ ●$\}$

$K = gfp(f)$

1. K is a fix point



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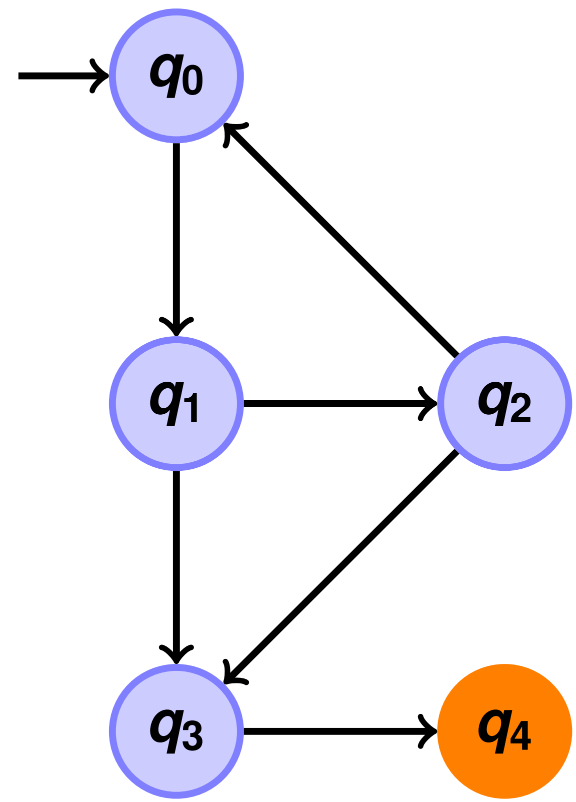


2. K is the greatest fix point

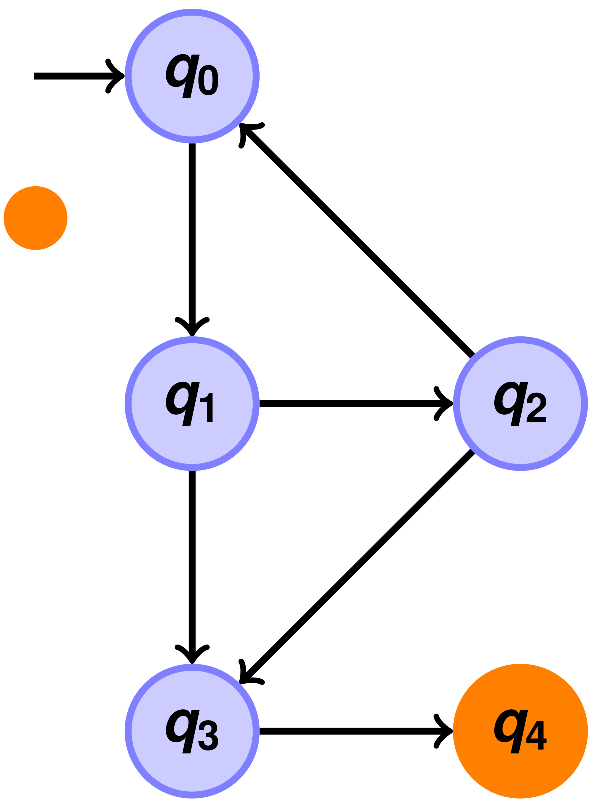
Example – 2

$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$ $K = \{q \mid q \models \mathrm{E\,G}\neg$ ●$\}$

$K = gfp(f)$

1. K is a fix point

$q \in K$:
- $q \models \mathrm{EG}\ \neg$ ●
- $q \to q' \xrightarrow{*} \cdots$, $q' \models \mathrm{EG}\ \neg$ ●
- hence $q' \in K$
- $q \in \delta^{-1}(K), q \models \neg$ ●
- hence $q \in f(K)$



2. K is the greatest fix point

Example – 2

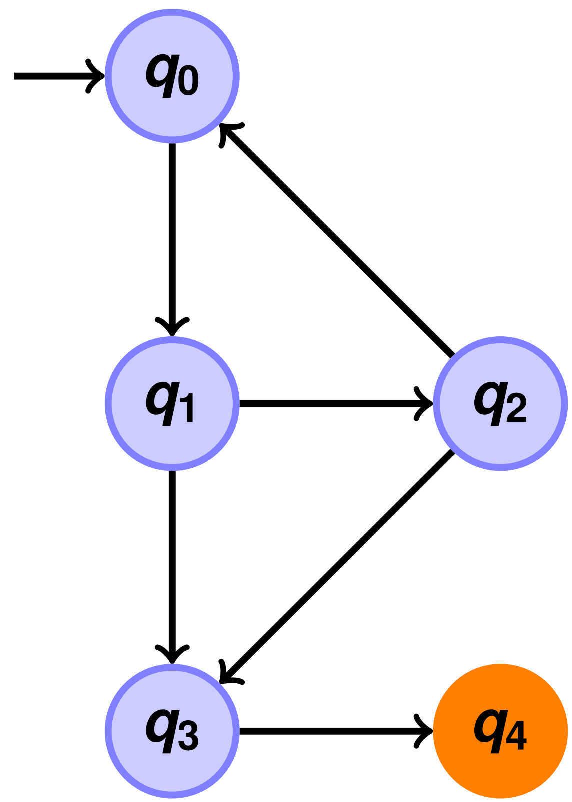
$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

$K = \{q \mid q \models \mathrm{E\,G}\neg$ ●$\}$

$K = gfp(f)$

1. K is a fix point

$q \in f(K) : \begin{cases} q \models \neg \text{●} \\ q \to q', q' \models \mathrm{EG}\ \neg \text{●} \\ \text{hence } q \models \mathrm{EG}\ \neg \text{●} \end{cases}$



2. K is the greatest fix point

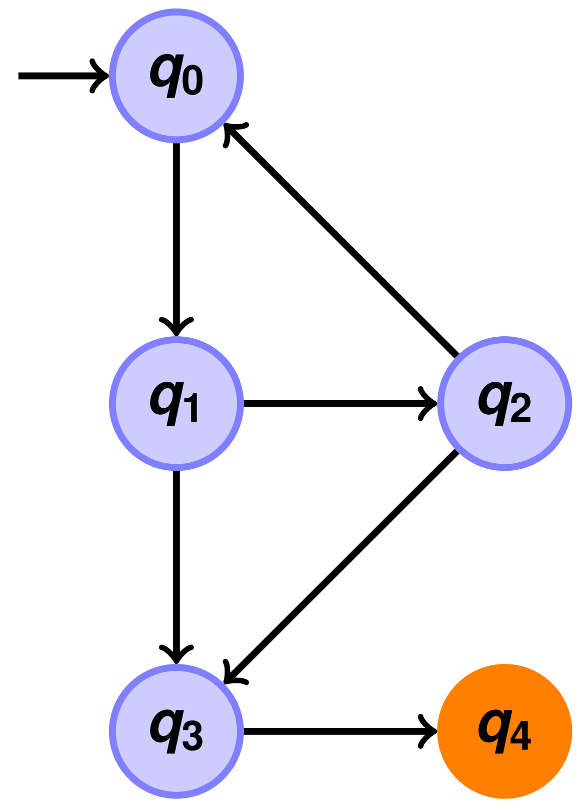
Example – 2

$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

$K = \{q \mid q \models \mathrm{E\,G}\neg$ ●$\}$

$K = gfp(f)$

1. K is a fix point **DONE**
2. K is the greatest fix point



Example – 2

$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

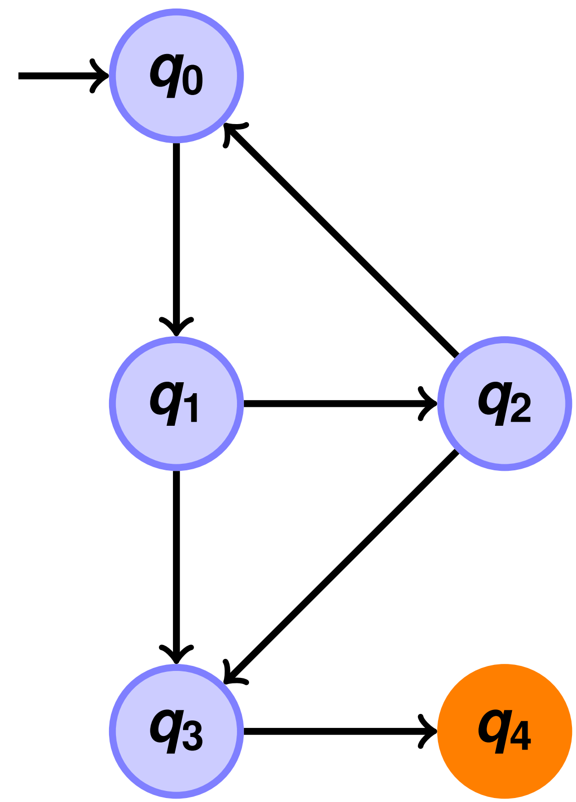
$K = \{q \mid q \models \mathrm{E\,G}\neg$ ●$\}$

$K = gfp(f)$

1. K is a fix point **DONE**
2. K is the greatest fix point

   $Y = f(Y)$

   Prove $q \in Y \implies q \in K$



Example – 2

$f(S) = (\neg \bullet) \cap \delta^{-1}(S)$ $K = \{q \mid q \models \mathrm{E\,G} \neg \bullet\}$

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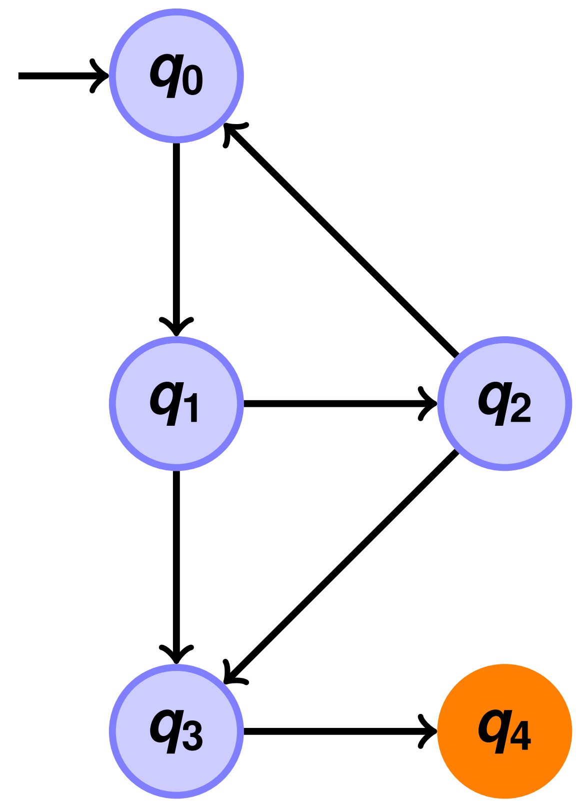
$Y = f(Y)$
Prove $q \in Y \implies q \in K$

- $q \in Y$



Example – 2

$\boldsymbol{f(S)} = (\neg$ ●$) \cap \delta^{-1}(\boldsymbol{S})$

$\boldsymbol{K} = \{\boldsymbol{q} \mid \boldsymbol{q} \models \mathrm{E\,G}\neg$ ●$\}$

$\boldsymbol{K = gfp(f)}$

1. $\boldsymbol{K}$ is a fix point **DONE**
2. $\boldsymbol{K}$ is the greatest fix point

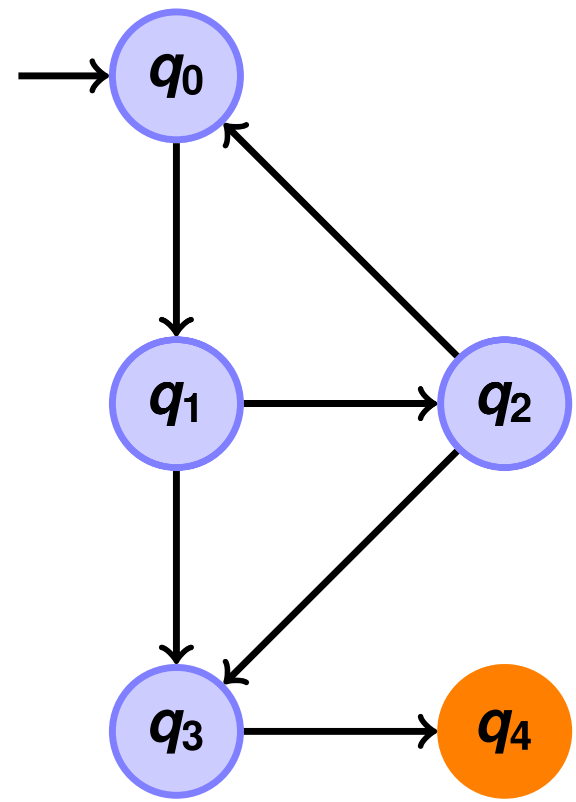
$\boldsymbol{Y = f(Y)}$

Prove $\boldsymbol{q} \in \boldsymbol{Y} \implies \boldsymbol{q} \in \boldsymbol{K}$

●

$\boldsymbol{q} \in \boldsymbol{Y}$

$\boldsymbol{q} \in \boldsymbol{f(Y)}$



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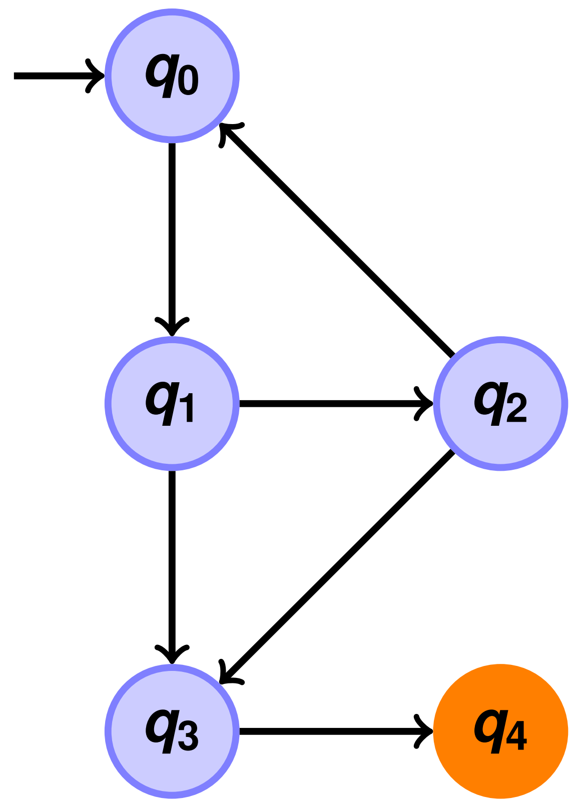
$q \models \neg$ ●

●

$q \in Y$

$q \in f(Y)$



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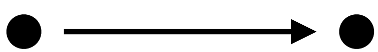
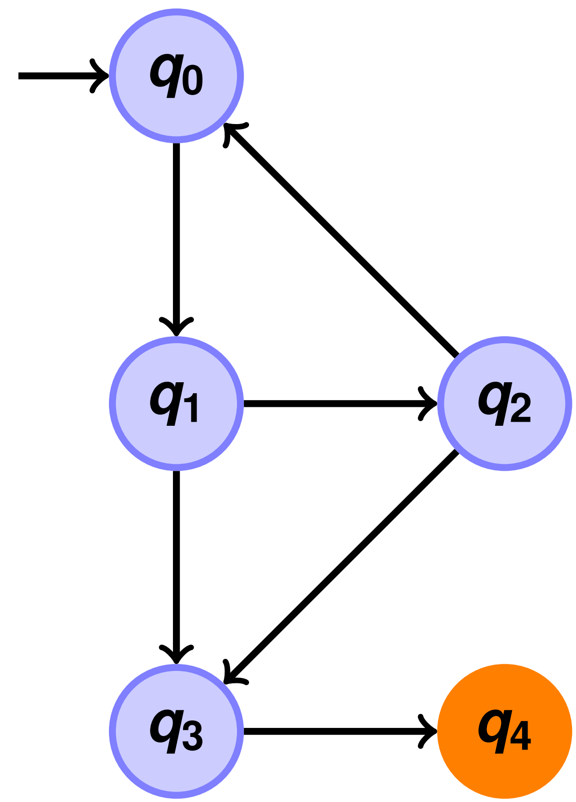


$q \in Y$ $q' \in Y$

$q \in f(Y)$



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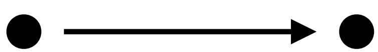

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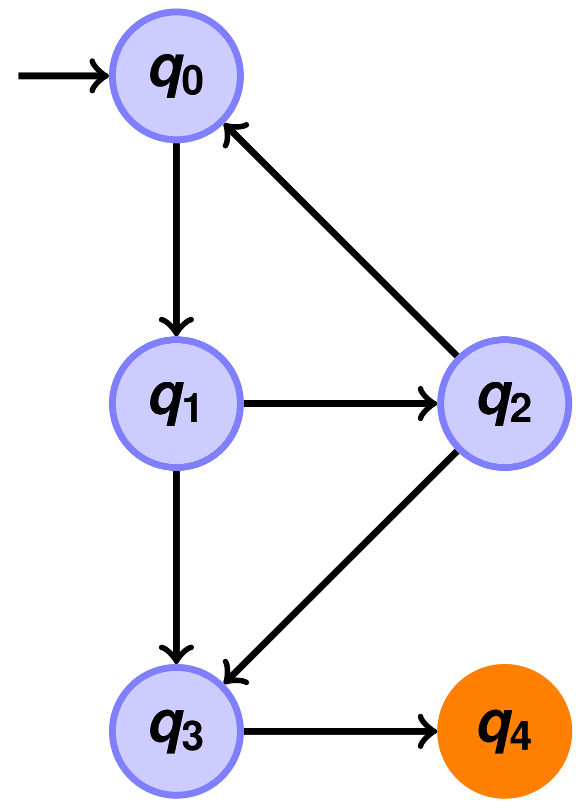

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$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

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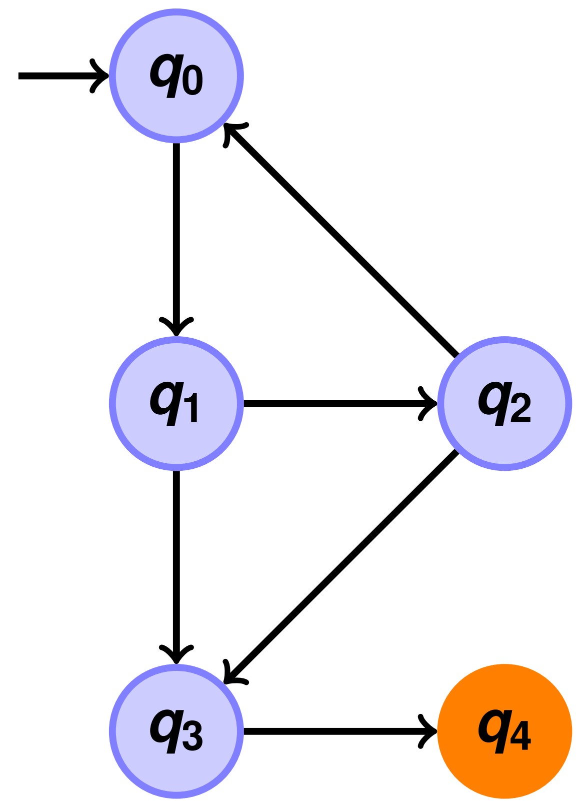
$q \models \neg$ ● $\quad q' \models \neg$ ●

● ⟶ ● ⟶ ●

$q \in Y \quad q' \in Y \quad q'' \in Y$

$q \in f(Y) \quad q' \in f(Y)$



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$\bullet \longrightarrow \bullet \longrightarrow \bullet$

$q \in Y \quad q' \in Y \quad q'' \in Y$

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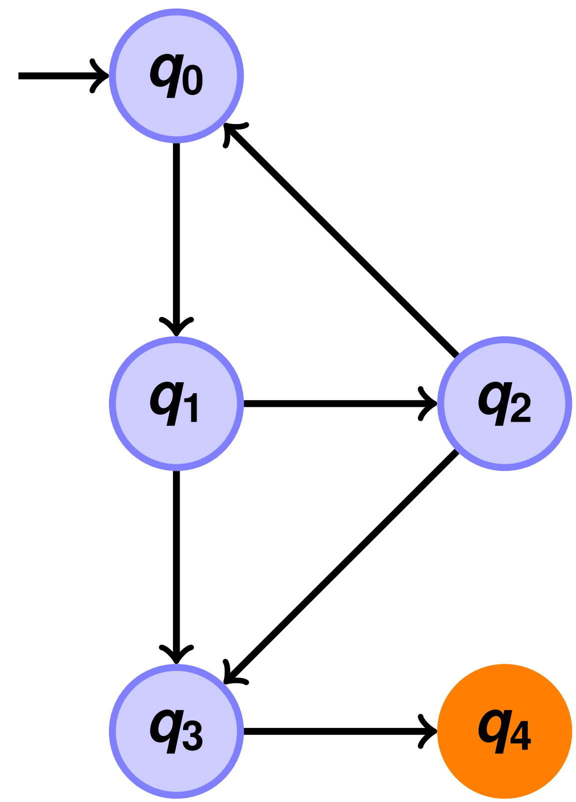

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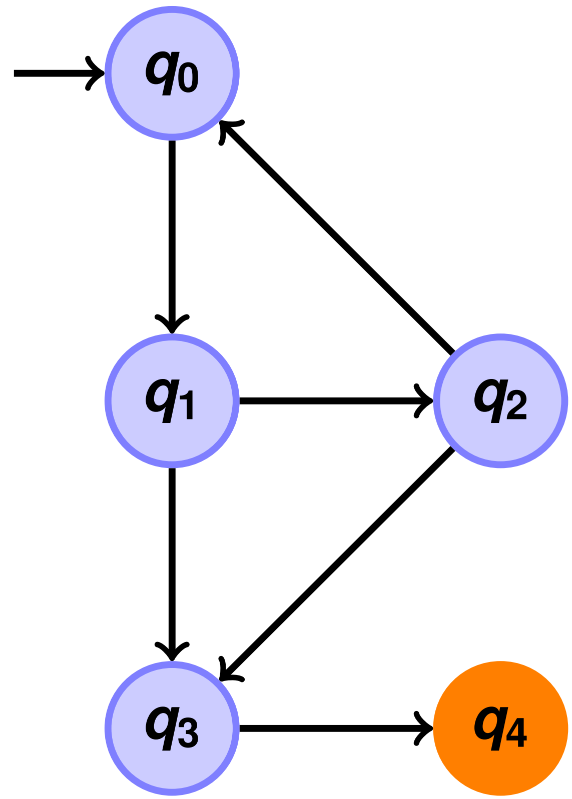


$q \models \neg$ ● $q' \models \neg$ ● $q'' \models \neg$ ●

● ⟶ ● ⟶ ● ....

$q \in Y$ $q' \in Y$ $q'' \in Y$

$q \in f(Y)$ $q' \in f(Y)$ $q'' \in f(Y)$

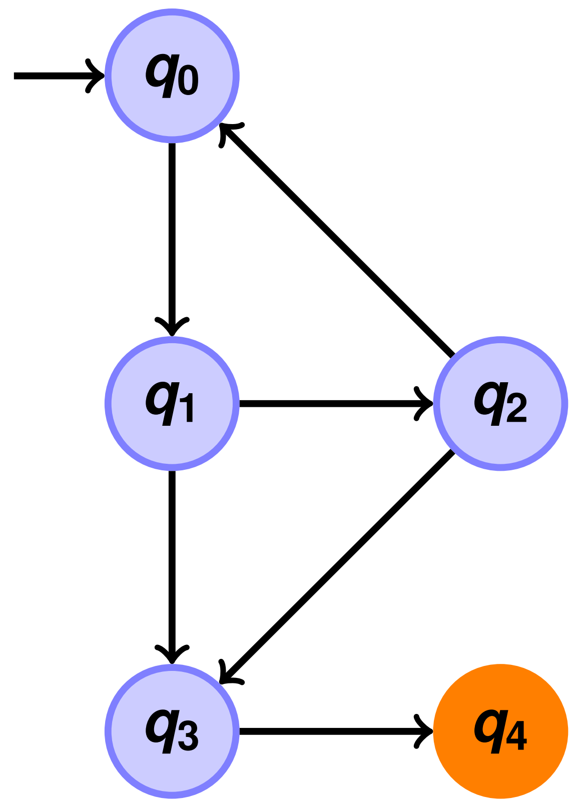
Example – 2

$f(S) = (\neg$ ●$) \cap \delta^{-1}(S)$

$K = \{q \mid q \models \mathrm{E\,G}\neg$ ●$\}$

$K = gfp(f)$

$K = f^k(\{q_0, q_1, q_2, q_3, q_4\})$



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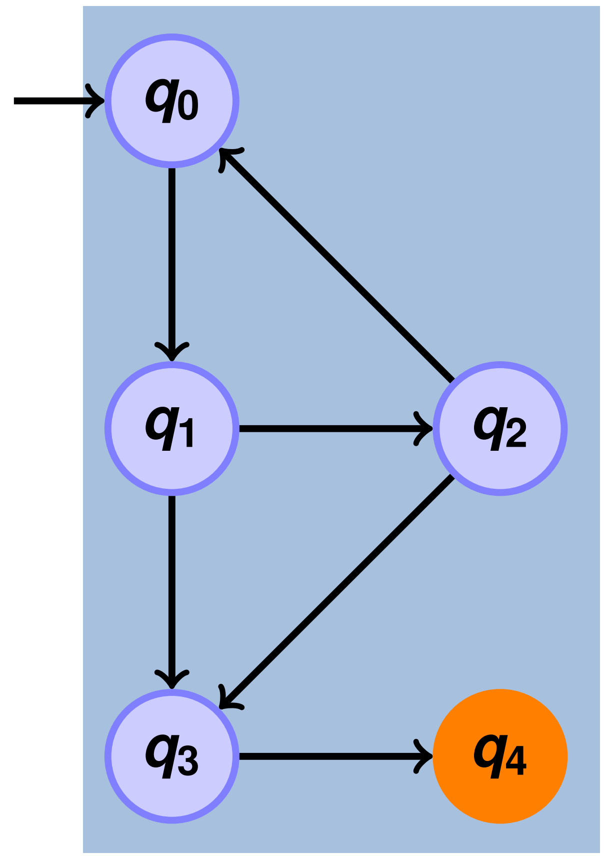

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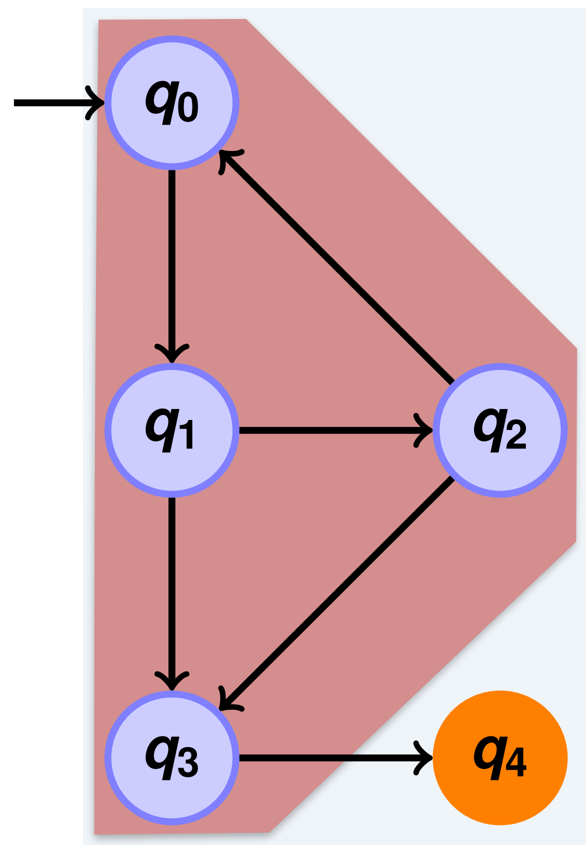

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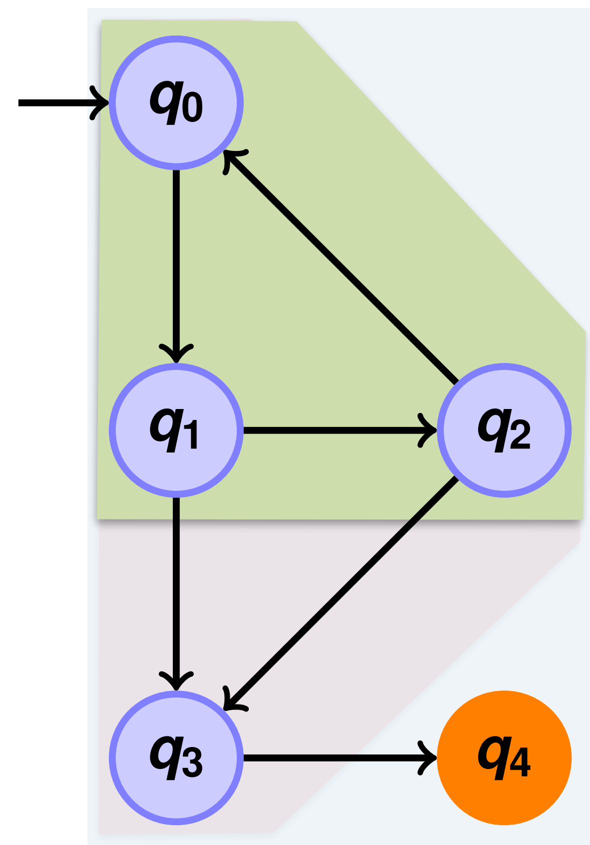

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$S_0 = \{q_0, q_1, q_2, q_3, q_4\}$
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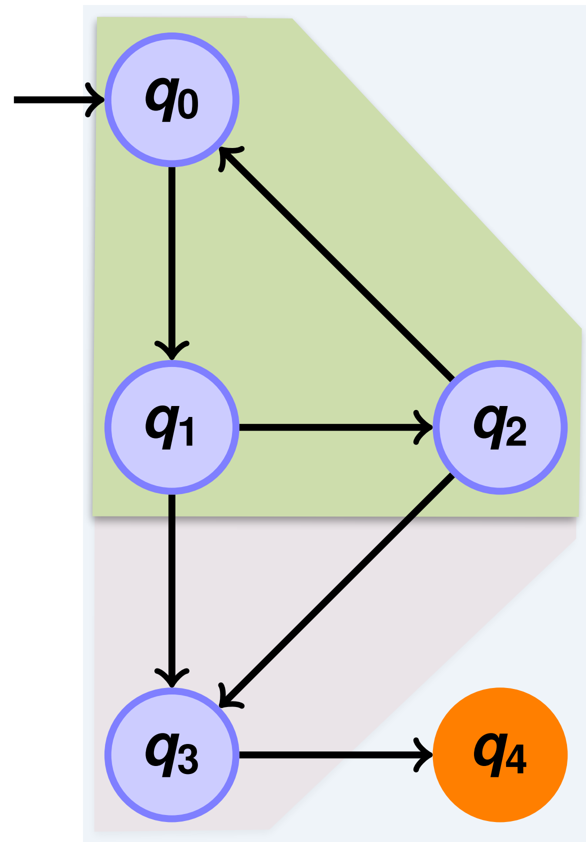


Fix Point Characterisation of CTL

CTL Syntax

CTL Syntax

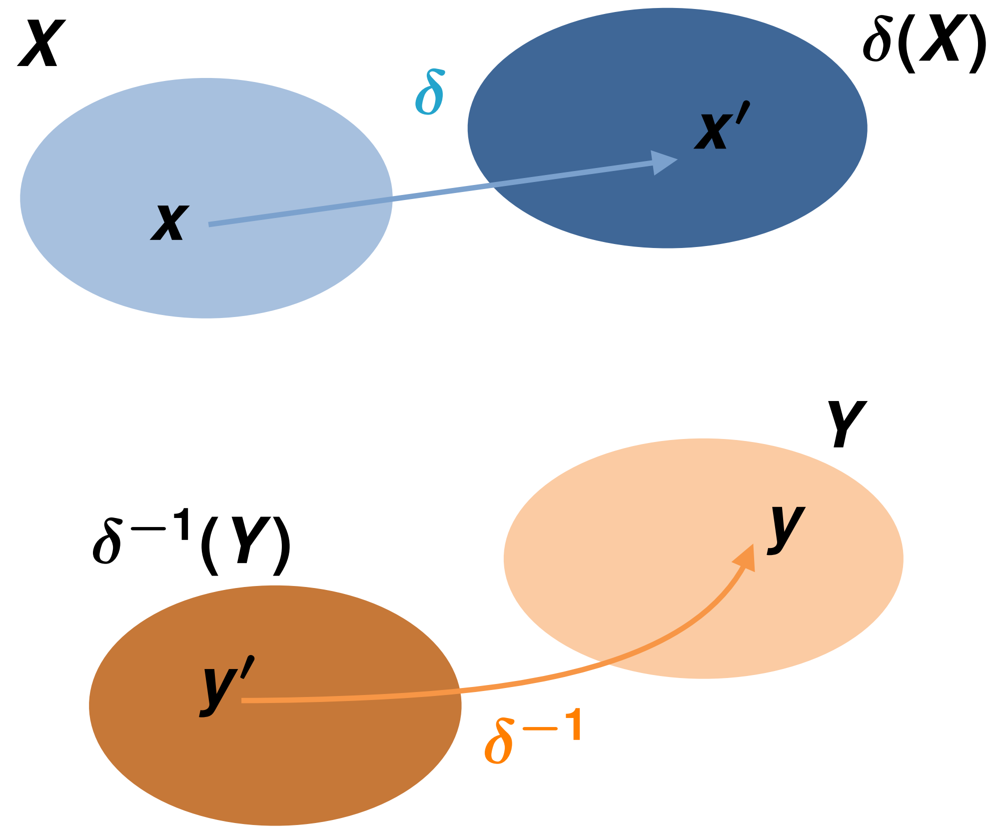
- True is in CTL
- $\boldsymbol{x} \in \mathcal{P}$ is in CTL
- $\boldsymbol{P}, \boldsymbol{Q} \in$ CTL, $\neg \boldsymbol{P} \in$ CTL and $\boldsymbol{P} \wedge \boldsymbol{Q} \in$ CTL
- E X φ is in CTL if φ is in CTL
- A X φ is in CTL if φ is in CTL
- A (φ_1 UNTIL φ_2) is in CTL if $\varphi_1, \varphi_2 \in$ CTL,
- E (φ_1 UNTIL φ_2) is in CTL if $\varphi_1, \varphi_2 \in$ CTL,

Successors/Predecessors (!)

A labeled automaton $A = (Q, q_0, \Sigma, \delta, L)$

$$\delta(X) = \bigcup_{x \in X} \{x' \mid x' \in \delta(x)\}$$

$$\delta^{-1}(Y) = \bigcup_{y \in Y} \{y' \mid y \in \delta(y')\}$$



CTL Symbolic Semantics (1)

Given $\varphi \in \boldsymbol{CTL}$

$[\![\varphi]\!] = \{s \mid s \models \varphi\}$

- $[\![True]\!] = Q$
- $p \in \mathcal{P}$, $[\![p]\!] = L^{-1}(p)$
- $[\![\neg P]\!] = Q \setminus [\![P]\!]$
- $[\![P \wedge Q]\!] = [\![P]\!] \cap [\![Q]\!]$
- $[\![\mathrm{E\,X}\ \varphi]\!] = \delta^{-1}([\![\varphi]\!])$
- $[\![\mathrm{A\,X}\ \varphi]\!] = Q \setminus \left(\delta^{-1}(Q \setminus [\![\varphi]\!])\right)$

CTL Symbolic Semantics (2)

$$\text{E}\,(\varphi_1 \text{ UNTIL } \varphi_2) \equiv \varphi_2 \vee \left(\varphi_1 \wedge \text{EX E}\,(\varphi_1 \text{ UNTIL } \varphi_2)\right)$$

$$s \models \text{E}\,(\varphi_1 \text{ UNTIL } \varphi_2) \iff \begin{cases} s \models \varphi_2 \text{ OR} \\ s \models \varphi_1 \wedge s \models \text{EX E}\,(\varphi_1 \text{ UNTIL } \varphi_2) \end{cases}$$

$$f_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(S) = [\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap \delta^{-1}(S)\right)$$

CTL Symbolic Semantics (2)

$$\text{E}\,(\varphi_1 \text{ UNTIL } \varphi_2) \equiv \varphi_2 \vee \big(\varphi_1 \wedge \text{EX E}\,(\varphi_1 \text{ UNTIL } \varphi_2)\big)$$

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$$f_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(S) = [\![\varphi_2]\!] \cup \big([\![\varphi_1]\!] \cap \delta^{-1}(S)\big)$$

$$[\![\text{E}\,(\varphi_1 \text{ UNTIL } \varphi_2)]\!] = \mathit{lfp}(f_{E(\varphi_1 \text{ UNTIL } \varphi_2)})$$

Proof

$f_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(S) = [\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap \delta^{-1}(S)\right)$

$[\![\text{E } (\varphi_1 \text{ UNTIL } \varphi_2)]\!] = \mathit{lfp}(f_{E(\varphi_1 \text{ UNTIL } \varphi_2)})$

to be proved!

$E_0 = [\![\text{E } (\varphi_1 \text{ UNTIL } \varphi_2)]\!]$

1. E_0 is a fix point $\quad f(E_0) \subseteq E_0$ and $E_0 \subseteq f(E_0)$

Proof

$f_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(S) = [\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap \delta^{-1}(S)\right)$

$[\![E\ (\varphi_1 \text{ UNTIL } \varphi_2)]\!] = lfp(f_{E(\varphi_1 \text{ UNTIL } \varphi_2)})$

to be proved!

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$[\![\text{E } (\varphi_1 \text{ UNTIL } \varphi_2)]\!] = lfp(f_{E(\varphi_1 \text{ UNTIL } \varphi_2)})$ **DONE**

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$s' \in Y$ and $s \in f(Y) \subseteq Y$

Effective Computation

$f_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(S) = [\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap \delta^{-1}(S)\right)$

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$(L, \leq)$ a finite lattice

$f : L \rightarrow L$ is monotonic

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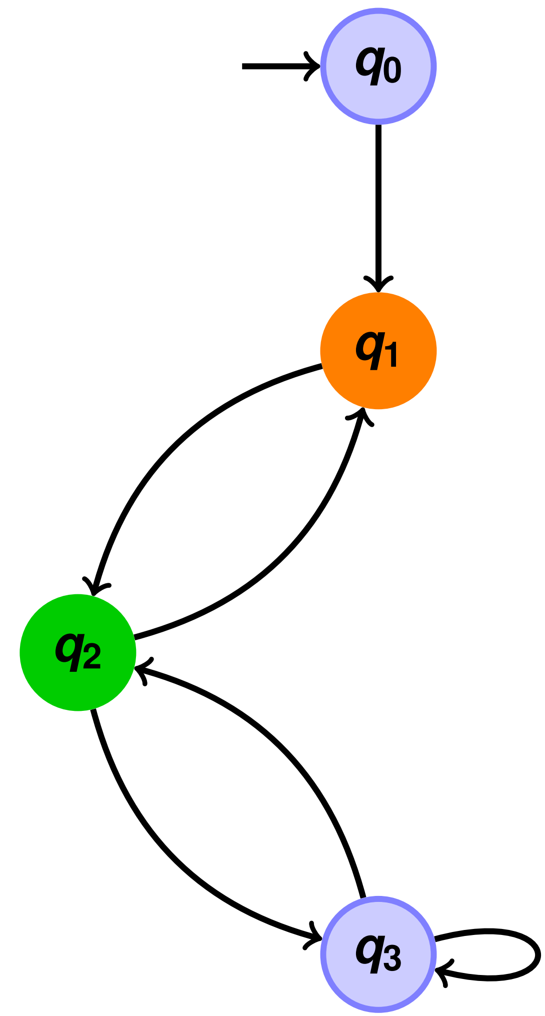
$[\![E(\varphi_1 \text{ UNTIL } \varphi_2)]\!] = f^k{}_{E(\varphi_1 \text{ UNTIL } \varphi_2)}(\emptyset),\ k \leq |Q|$

Example

φ = E ● UNTIL ●

$f(S) = [\![●]\!] \cup ([\![●]\!] \cap \delta^{-1}(S))$

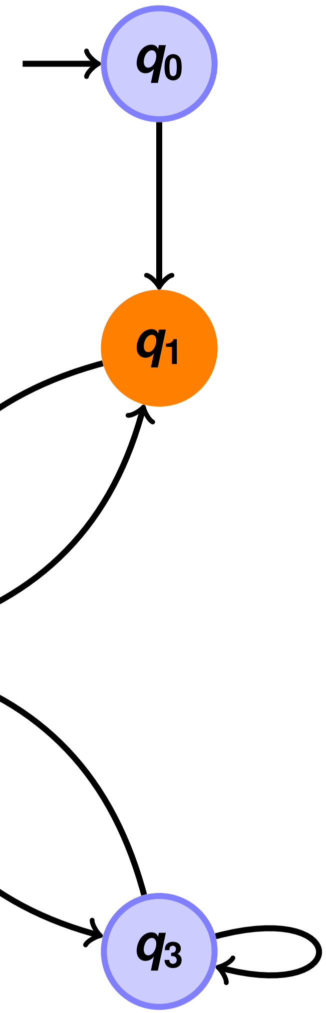


Example

φ = E 🟠 UNTIL 🟢

***f*(*S*) = ⟦ 🟢 ⟧ ∪ (⟦ 🟠 ⟧ ∩ δ^{-1}(*S*))**

Compute ***lfp*(*f*)**



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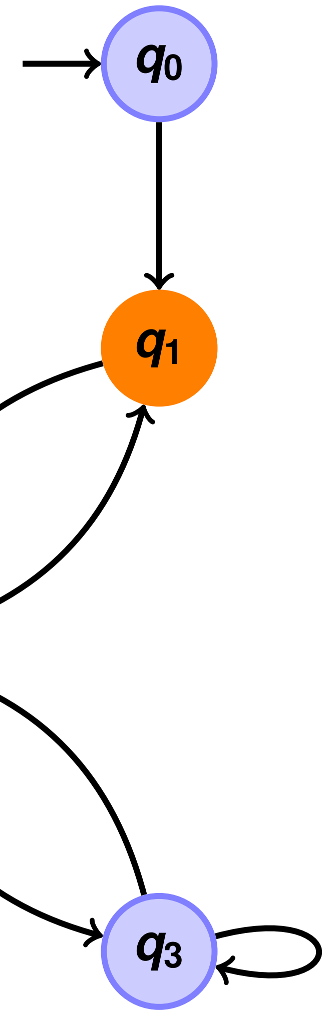

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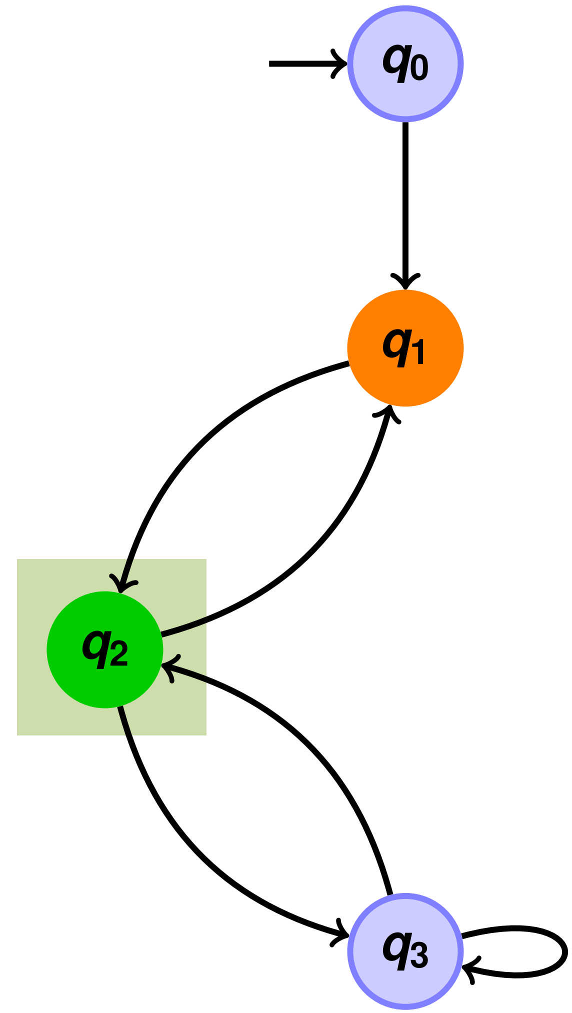

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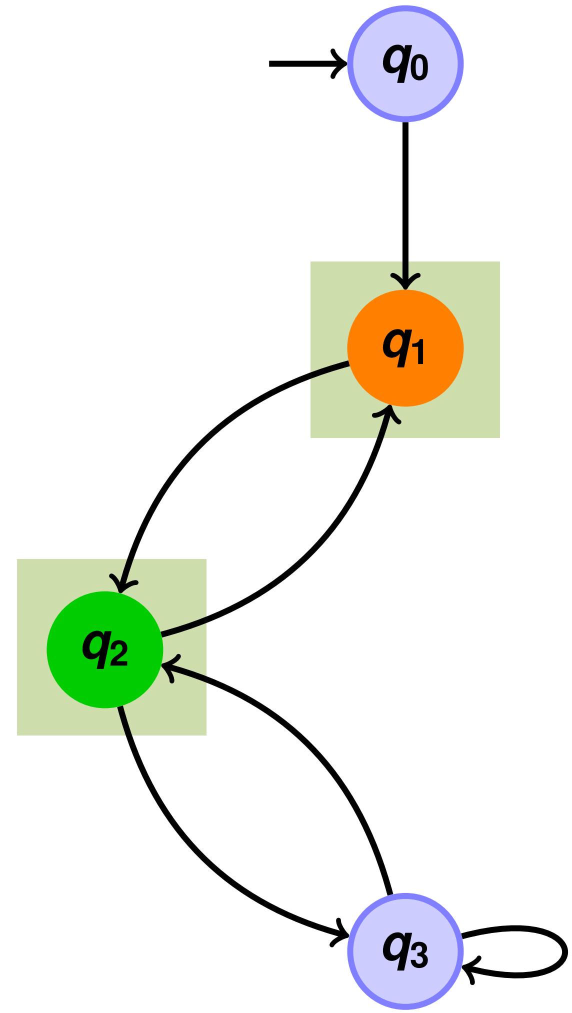

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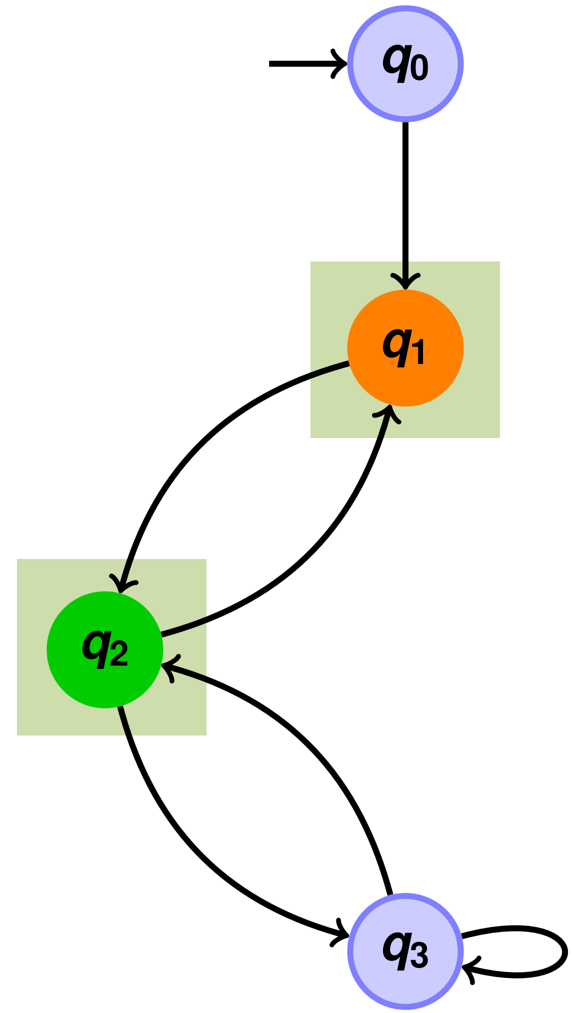

What is ***gfp*(*f*)**?

Other Operators

$lfp(f(X)) \equiv \mu X.f(X)$ $gfp(f(X)) \equiv \nu X.f(X)$

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$lfp(f(X)) \equiv \mu X.f(X)$ $gfp(f(X)) \equiv \nu X.f(X)$

- $[\![E(\varphi_1 \text{UNTIL} \varphi_2)]\!] = \mu X.[\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap \delta^{-1}(X)\right)$
- $[\![A(\varphi_1 \text{UNTIL} \varphi_2)]\!] = \mu X.[\![\varphi_2]\!] \cup \left([\![\varphi_1]\!] \cap (Q \setminus \delta^{-1}(Q \setminus X))\right)$
- $[\![EF\varphi]\!] = \mu X.[\![\varphi]\!] \cup \delta^{-1}(X)$
- $[\![EG\varphi]\!] = \nu X.[\![\varphi]\!] \cap \delta^{-1}(X)$
- $[\![AG\varphi]\!] = \nu X.[\![\varphi]\!] \cap (Q \setminus \delta^{-1}(Q \setminus X))$
- $[\![AF\varphi]\!] = \mu X.[\![\varphi]\!] \cup (Q \setminus \delta^{-1}(Q \setminus X))$

EF ●

$[\![\mathrm{EF}\varphi]\!] = \mu \boldsymbol{X}.[\![\varphi]\!] \cup \delta^{-1}(\boldsymbol{X})$

$\boldsymbol{f}(\boldsymbol{X}) = [\![$ ● $]\!] \cup \delta^{-1}(\boldsymbol{X})$

1. What is $[\![\mathrm{EF}$ ●$]\!]$?

2. What is the greatest fix point of ***f***?

3. What is the least fix point of ***f***?